STATIC PULL-IN INSTABILITY ANALYSIS OF A MICROCANTILEVER UNDER UNIFORMLY DISTRIBUTED LOADING AND ELECTROSTATIC INTERACTION FORCE USING SHOOTING METHOD

PHÂN TÍCH TĨNH CHO DẠNG MẤT ỔN ĐỊNH KÉO VÀO CỦA MỘT DẦM CÔNG-XÔN KÍCH THƯỚC MICRO CHỊU TẢI PHÂN BỐ ĐỀU VÀ LỰC TƯƠNG TÁC TĨNH ĐIỆN BẰNG PHƯƠNG PHÁP BẮN

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ABSTRACT

In this study, using the shooting method, the static pull-in instability phenomenon of a microcantilever beam under the effect of uniformly distributed loading and electrostatic interaction force is analyzed within the framework of the Euler-Bernoulli beam theory associated with the modified couple stress theory. The shooting method is a numerical treatment applied to the boundary value problem using approaches of numerical methods for initial condition problems. For the microcantilever beam under consideration, the shooting method is implemented based on the Runge-Kutta method combined with the Newton-Raphson iteration method. The profit of the present approach is to take advantage of the accuracy of the well-known Runge-Kutta algorithm and the Newton-Raphson method for algebraic equations with fast convergence. Because the governing equation of the beam is nonlinear, a highly accurate method becomes important in determining the pull-in characteristics of micro-electro-mechanical systems (MEMS). The static response of the microbeam is investigated in detail. The pull-in curves are shown to be a good agreement in comparison with those obtained from the finite difference method and experimental data.

Keywords: *Modified couple stress theory; Microcantilever; Shooting method; Electrostatic force; pull-in instability.*

TÓM TẮT

Trong nghiên cứu này, các tác giả sử dụng phương pháp bắn để phân tích hiện tượng mất ổn định tĩnh dạng kéo vào (pull-in) cho một dầm công-xôn kích thước micro dưới tác dụng của lực ngoài có phân bố đều và lực tương tác tĩnh điện trong khuôn khổ lý thuyết dầm Euler-Bernoulli và lý thuyết ứng suất ngẫu. Phương pháp bắn là một phương pháp số để xử lý bài toán điều kiện biên dựa vào phương pháp số cho bài toán điều kiện đầu. Với bài toán đang xét, phương pháp bắn ☞

được thực hiện thông qua việc áp dụng thuật toán Runge-Kutta bậc bốn đã biết và phương pháp lặp Newton-Raphson. Ưu điểm của cách tiếp cận hiện tại đó là sự kết hợp giữa ưu điểm về độ chính xác của phương pháp số Runge-Kutta và tốc độ hội tụ nhanh của phương pháp Newton-Raphson. Bởi vì phương trình chuyển động của dầm có dạng phi tuyến, việc tìm một phương pháp chính xác cao để đánh giá sự mất ổn định kéo vào là một bài toán có ý nghĩa. Kết quả tính toán số được phân tích chi tiết cho các trường hợp hệ với các tham số khác nhau. Các tác giả cũng chỉ ra rằng kết quả thu được là tương đối phù hợp với kết quả đã có trước đây sử dụng phương pháp sai phân hữu hạn và phương pháp thực nghiệm.

Từ khóa: *Lý thuyết ứng suất ngẫu cải tiến; Dầm micro công-xôn; Phương pháp bắn; Lực tĩnh điện; Hiện tượng mất ổn định tĩnh dạng kéo vào.*

1. INTRODUCTION

Micro-electro-mechanical systems (MEMS) play an important role in research and development for modern applications today. MEMS are integrated in devices for applications such as sensors, actuators, microswitches, micropumps, micromirrors, etc. [1,2]. MEMS systems involve microscale structures that are often made of constituent components such as beams/plates with different material/geometrical configurations placed in a connection space with substrates through electrostatic interactions. When a voltage is applied, an electrostatic interaction occurs to make the beams/plates deform and be attracted to the substrate direction. The beam/plate may touch the substrate as the voltage reaches a certain critical value. This is known as pull-in instability phenomenon [3,4,5]. This phenomenon is generally undesirable because it can damage or destroy the MEMS structure. Pull-in instability is studied in both theoretical and experimental aspects using different methods and approaches [6]. For the theoretical research aspect, beam/plate models are developed to better predict static and dynamic characteristics of system responses and pull-in parameters. Rhaeifard et al. [7] used the Euler-Bernoulli beam model associated with the modified couple stress theory to study static pull-in instability of a microcantilever beam. Those authors have shown that the couple stress theory can remove an existing gap between the experimental observations and classical theory. Taati and Sina [8] investigated an electrostatically actuated functionally graded micro-beams based on the modified strain gradient theory. Their results provide a good reference for pull-in voltages where the pull-in points predicted by the modified strain gradient theory are very close to those obtained from the experiments. Recently, a model of nonlinear dynamics [9] has been developed for analyzing the pull-in instability phenomena of a microcantilever beam based on the modified couple stress theory. Dai and Wang [9] have shown that the presence of geometric nonlinearities of beam model and length-scale parameter can significantly change the pull-in band of frequencies. Such a dynamic model helps us understand further the nature of pull-in phenomenon in MEMS systems.

In practice, the MEMS systems often operate in environments with complex interactions, including external excitations which affect the response characteristics of the system. To better describe the pull-in phenomenon, in this study, the authors have proposed to investigate the influence of external loads on the system response using a numerical approach based on the shooting method. In the framework of the present study, the external load is assumed to be uniformly distributed along the length of the beam, acting on a beam modeled using the modified couple stress theory. The influence of external loads can lead to a shift of the pull-in curve. This shift is a new observation and has been analyzed in this study.

2. GOVERNING EQUATION OF MICROCANTILEVER AND SOLUTION TREATMENT METHOD

We consider a microcantilever beam with the length L, cross-section bxh, under a uniformly distributed loading q_0 and electrostatic interaction force q_e, as illustrated in Fig. 1. The Euler-Bernoulli beam theory associated with the modified couple stress theory is used to model the beam under consideration. The beam material is assumed to be linear and isotropic. The governing equation for beam is given by [7]:

$$\left(EI + \mu A l^2\right)\frac{d^4 w}{dx^4} = q_e + q_0 \qquad (1)$$

where w = w(x) is the deflection; E, I are the Young's modulus and the second-order moment of inertial of cross-section, respectively; μ is the shear modulus; A is area of cross-section; l is a length scale parameter corresponding to the modified couple stress theory.

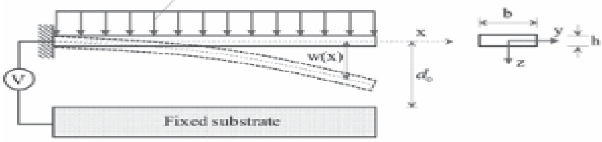

Figure 1. Model of a microcantilever beam under the uniformly distributed loading and electrostatic interaction force

The electrostatic force q_e is obtained from the electrostatic interaction between surfaces of the beam and a fixed substrate when a DC voltage V is actuated. In this study, the fringing field effect is included in the electrostatic interaction, and therefore, the expression for q_e takes the following form [10]:

$$q_e = \frac{1}{2}\frac{\varepsilon_0 b V^2}{\left(d_0 - w\right)^2}\left(1 + 0.65\frac{d_0 - w}{b}\right) \qquad (2)$$

where $\varepsilon_0 = 8.854 \times 10^{-12}$ (F/m) is the vacuum permittivity coefficient; d_0 is the gap distance between the beam and fixed substrate. In the expression (2), the second term represents the fringing field effect. For the cantilever beam, the boundary conditions are given by:

$$w(0) = \frac{dw}{dx}(0) = \frac{d^2 w}{dx^2}(L) = \frac{d^3 w}{dx^3}(L) = 0 \qquad (3)$$

We introduce the following quantities:

$$\xi = \frac{x}{L},\ y = \frac{w}{d_0},\ \alpha = 1 + \frac{12\mu}{E}\frac{1}{\left(h/l\right)^2},\ \beta = \frac{\varepsilon_0 b L^4}{2EId_0^3},\ \gamma = \frac{q_0 L^4}{EId_0} \qquad (4)$$

Substituting Eq. (2) into Eq. (1) and using new notations in (4), Eq. (1) is transformed into the following form of dimensionless deflection y = y(ξ) with $0 \le \xi \le 1$:

$$y^{(4)}(\xi) = \frac{\beta V^2}{\alpha}\frac{1}{\left(1 - y\right)^2}\left(1 + \frac{0.65}{r_{bd}}\left(1 - y\right)\right) + \frac{\gamma}{\alpha} \qquad (5)$$

Where $y^{(4)}$ denotes the fourth-order derivative of dimensionless y(ξ), r_{bd} is the ratio of the beam's width b to the gap distance d_0, i.e. $r_{bd} = b/d_0$. The boundary conditions for Eq. (5) are obtained from Eq. (3) as follows:

$$y(0) = y'(0) = y''(1) = y'''(1) = 0 \quad (6)$$

Where the prime denotes the derivative corresponding to the variable ξ. By introducing new state variables $y_1 = y(\xi),\ y_2 = y'(\xi),\ y_3 = y''(\xi),\ y_4 = y'''(\xi)$ Eq. (5) becomes a system of state variable equations:

$$y_1' = y_2,\ y_2' = y_3,\ y_3' = y_4$$
$$y_4' = \frac{\beta V^2}{\alpha}\frac{1}{(1-y_1)^2}\left(1+\frac{0.65}{r_{bd}}(1-y_1)\right)+\frac{\gamma}{\alpha} \quad (7)$$

With boundary conditions:

$$y_1(0) = y_2(0) = y_3(1) = y_4(1) = 0 \quad (8)$$

In this study, our approach is to apply a method, called shooting method [11,12], to solve the boundary condition problem (7) based on the well-known fourth-order Runge-Kutta method of initial condition problems. The idea of the shooting method is to find solution of the initial value problem for different values of initial conditions until the desired solution satisfies the boundary conditions of the boundary value problem. To that end, initial conditions for Eq. (7) are introduced as follows:

$$y_1(0) = y_2(0) = 0,\ y_3(0) = y_{30},\ y_4(0) = y_{40} \quad (9)$$

Where y_{30}, y_{40} are two unknowns and must be determined from the original boundary conditions (8). From Eq. (7), the solution variables y_1, y_2, y_3, y_4 have the forms that depend on two parameters y_{30}, y_{40}:

$$y_1 = y_1(\xi, y_{30}, y_{40}),\ y_2 = y_2(\xi, y_{30}, y_{40}),\ y_3 = y_3(\xi, y_{30}, y_{40}),\ y_4 = y_4(\xi, y_{30}, y_{40}) \quad (10)$$

Using Eq. (10), two last conditions in Eq. (9) lead to the following system of two variables y_{30}, y_{40}:

$$\begin{aligned} Y_3(y_{30}, y_{40}) &:= y_3(1, y_{30}, y_{40}) = 0, \\ Y_4(y_{30}, y_{40}) &:= y_4(1, y_{30}, y_{40}) = 0 \end{aligned} \quad (11)$$

The algebraic system (11) of two unknowns y_{30}, y_{40} is solved by the Newton-Raphson method with the following iteration:

$$\begin{bmatrix} y_{30}^{(n+1)} \\ y_{40}^{(n+1)} \end{bmatrix} = \begin{bmatrix} y_{30}^{(n)} \\ y_{40}^{(n)} \end{bmatrix} - \begin{bmatrix} \frac{\partial Y_3}{y_{30}} & \frac{\partial Y_3}{y_{40}} \\ \frac{\partial Y_4}{y_{30}} & \frac{\partial Y_4}{y_{40}} \end{bmatrix}^{-1}_{\left(y_{30}^{(n)}, y_{40}^{(n)}\right)} \begin{bmatrix} Y_3 \\ Y_4 \end{bmatrix}_{\left(y_{30}^{(n)}, y_{40}^{(n)}\right)} \quad (12)$$

The iteration is convergent if the condition $\left\|\left(y_{30}^{(n)}, y_{40}^{(n)}\right)\right\| \le \varepsilon$ is satisfied for an arbitrary small tolerance ε ($\varepsilon = 10^{-12}$ in our computation). In the next section, using the shooting method, we explore unique characteristics of response related to pull-in instability phenomena with different parameters of the microcantilever beam system.

3. NUMERICAL RESULTS AND DISCUSSIONS

We have carried out numerical calculations to find solutions of dimensionless deflection y(ξ) and the corresponding response at tip, i.e. y(1), of the beam using the shooting method for the boundary value problem (7) associated with the iteration process (12). The beam's material and geometrical parameters are given in Tab. 1. The gap distance is kept at d_0 = 1.05 (μm). The ratio of the beam's width b to the gap distance d_0 is computed to be r_{bd} = 47.9190. The ratio h/l is selected as h/l = 4.96.

Table 1. Material and geometrical parameters of the microcantilever beam

E (GPa)	μ (GPa)	v	L (μm)	h (μm)	b (μm)
169.2	65.8	0.239	75 ÷ 200	2.94	50

3.1. Pull-in instability of beam without uniformly distributed loading

In this subsection, our calculations are obtained in the case of absence of the uniformly distributed loading, i.e. $q_0 = 0$. The curves of dimensionless deflection or deflection ratio at tip $\xi = 1$ depending on the DC applied voltage V with different values of beam's length are portrayed in Fig. 2. The value of length L is selected from 75 (μm) to 200 (μm). It is seen that the beam deflection will increase as the applied voltage increases. If this voltage tends to a critical value, the deflection will change suddenly and, consequently, the beam contacts with the fixed substrate. This phenomenon is known as pull-in instability. The critical voltage is called the pull-in voltage, denoted as V_{PI}. For the beam's length L = 75 (μm), the obtained pull-in voltage is approximate to V_{PI} = 76.5 (V). Because in this case the length of beam is relatively short, it needs a large voltage to attain the pull-in value. This means that if the interval (0, V_{PI}) is considered as a working domain in which the device operates well and no collision appears, a wide range of V_{PI} will help the device become more flexible with the applied voltage. In design, however, the length of beam is limited to a certain value, the value of applied voltage can't be chosen arbitrarily. The determination of pull-in voltage therefore becomes important. For longer beams, the interval (0, V_{PI}) becomes narrower, for example, for L = 100 (μm), the pull-in voltage is found to be V_{PI} = 43 (V), that is smaller than V_{PI} = 76.5 (V) in the case L = 75 (μm). The curves of deflection ratio y exhibit a monotonically increasing property as the applied voltage increases. The slope of these curves reflects the rate of change of the beam deflection when the applied voltage changes. The longer length of beam leads to a larger slope of deflection. This means that the pull-in instability occurs "faster". This feature shows the voltage-dependent sensitivity of long beam in microscale devices.

Fig. 3 illustrates the dependence of pull-in voltage V_{PI} on the beam length L for different approaches to solve the problem under consideration. It is seen that when the length scale parameter l in Eq. (1) is set to be zero, the the governing equation (1) is reduced to the one corresponding to the classical Euler-Bernoulli beam theory. The curves of deflection ratio y are plotted for the present shooting method approach, the finite-difference method by Rahaeifard et al, experimental data by Osterberg, and classical beam theory. The obtained results show that our method is a good agreement with the experimental data in the case the length scale effect is taken into account for the model of cantilever beam at microscale. In comparison with the experimental data, the classical beam theory leads to an underestimate for the pull-in voltage point as the beam's length changes. As observed in Fig. 3, this lack has been dealt with and fulfilled when the modified couple stress theory is used to model the beam in the problem of electrostatic interaction at microscale.

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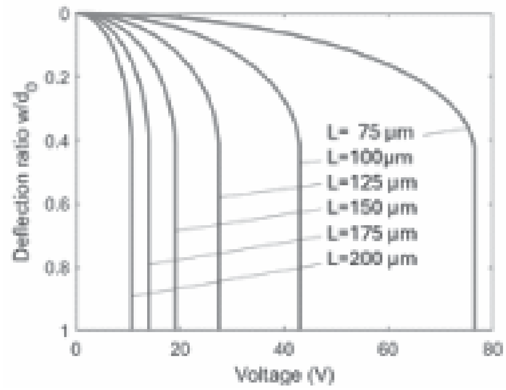

Figure 2. Graphs of deflection ratio $y = w/d_0$ versus the applied voltage with 6 cases of beam length: L = 75, 100, 125, 150, 175, 200 (µm)

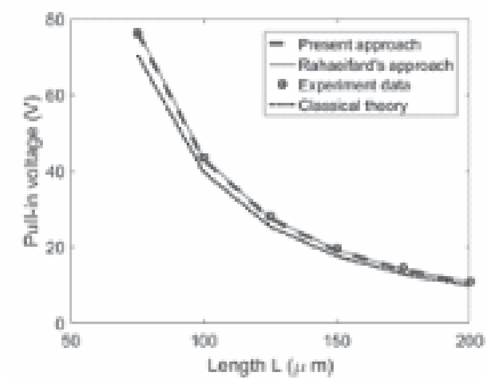

Figure 3. Plots of pull-in voltage V_{PI} versus length L using four methods: present approach, Rahaeifard's approach, experiment data, and classical Euler-Bernoulli beam theory.

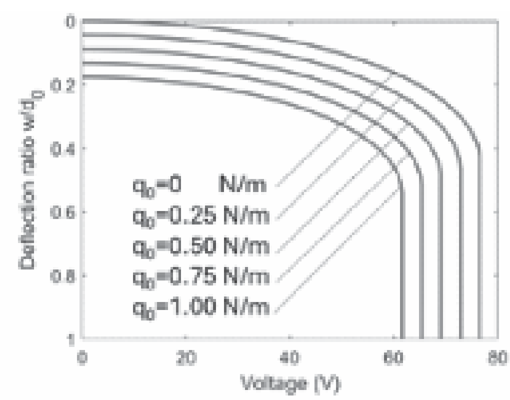

Figure 4. Graphs of deflection ratio w/d_0 versus the applied voltage V with different values q_0 = 0, 0.25, 0.5, 0.75, 1.0 (N/m).

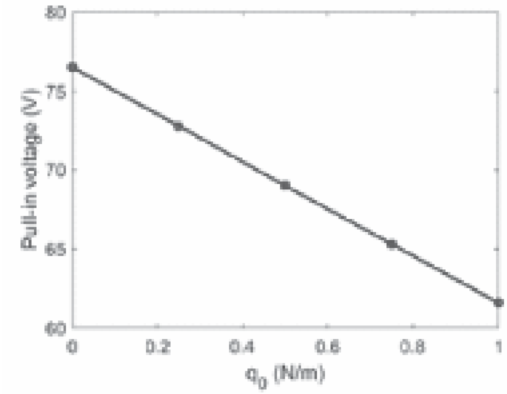

Figure 5. Dependence of the pull-in voltage on the uniformly distributed loading q_0

3.2. Augmentation of pull-in instability due to the external loading

The presence of uniformly distributed loading q_0 leads to the fact that there exists a deflected beam configuration before the DC voltage is applied. In this situation, the beam's tip has an initial deflection at V = 0(V). The material and geometrical parameters are the same as in the previous subsection. Numerical calculations are performed for the case of beam length L = 75 (µm) with different values of q_0, from 0 (N/m) to 1.0 (N/m). As learned in books on Strength of Materials, the maximum deflection of cantilever beam under the uniformly distributed loading q_0 is $q_0L^4/(8EI)$. Because the fixed substrate lies under the beam at a distance d_0, the loading q_0 must be satisfied the condition$q_0L^4/(8EI) < d_0$. This gives the resulting condition $q_0 < 8EId_0 /L^4$ for which the quantity $8EId_0 /L^4$ can be considered as a maximum value of external applied loading q_0 before the applied voltage V is actuated in system. The curves of deflection ratio $y = w/d_0$ for different values of q_0 are plotted in Fig. 4. Five values of q_0 are selected, q_0 = 0, 0.25, 0.50, 0.75, 1.0 (N/m). In the case q_0 = 0 (N/m), the deflection curve is the same as in Fig. 2, for which the pull-in voltage is found to be V_{PI} = 76.5 (V). The starting point of the curve is zero because at V = 0, q_0 = 0, the beam's tip is on initial state with zero deflection. The presence of loading q_0 generates a shift of pull-in curve in the increasing direction of deflection. This shift, however, makes the working voltage domain (0, V_{PI}) narrower. This feature exhibits an aspect on the augmentation of pull-in instability due to the added external loading. In real engineering applications, when operating in harsh and complex environments, external loads are always present, and therefore their influence on the pull-in characteristic must be taken into account. For a simple, uniformly distributed external loading model, the effect of external loading is reflected through the decreasing trend of the pull-in voltage. It is described as follows: since the beam has initial deflection due to an external load q_0, the tip of the beam is pushed to the substrate direction; as the voltage is activated, the tip of the beam will be pushed further to the substrate, but with a voltage smaller than that in the case without initial deflection. The pull-in voltage V_{PI} depending on the loading q_0 is portrayed in Fig. 5. It is observed that, the relation between the pull-in voltage V_{PI} and loading q_0 is a nearly linear dependence. The calculated slop of the curve $q_0 - V_{PI}$ is approximate to value 15 (Vm/N).

4. CONCLUSION

The pull-in instability is an inherent characteristic of micro-electro-mechanical systems (MEMS) actuated by applied voltage. Finding pull-in points is an important issue in the design and manufacturing process of MEMS systems. In this study, the characteristics of pull-in curves and corresponding pull-in voltage points were investigated using the shooting method based on the fourth-order Runge-Kutta algorithm combined with the Newton-Raphson iteration method. The following highlights are drawn from the numerical results of the microcantilever beam problem under consideration:

• The deflection at the tip of the beam increases in the increasing direction of applied voltage before the pull-in phenomenon occurs. For short beams, the pull-in voltage has a large value, and becomes smaller for longer beams. This reflects the fact that the beams with different lengths will operate within a certain limited voltage range.

• In the framework of the modified couple stress theory, numerical solutions of beam response obtained from our present approach show a good agreement with those received from experiment data.

• The augmentation of pull-in instability is described and analyzed for the first time, in a simple situation of external loadings in which the uniformly distributed loading is considered. The dependence of the pull-in voltage on the constant external loading is nearly linear.

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