MULTI-OBJECTIVE OPTIMIZATION OF A TWO-STAGE HELICAL GEARBOX WITH A SPLIT SLOW STAGE USING NSGA-II AND MARCOS: MINIMIZING FOOTPRINT AND MAXIMIZING EFFICIENCY

TỐI ƯU HÓA ĐA MỤC TIÊU HỘP SỐ BÁNH RĂNG TRỤ HAI CẤP VỚI CẤP CHẬM PHÂN ĐÔI SỬ DỤNG NSGA-II VÀ MARCOS: GIẢM THIỂU DIỆN TÍCH VÀ TỐI ĐA HÓA HIỆU SUẤT

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ABSTRACT

This study presents a hybrid multi-objective optimization framework that combines the Non-dominated Sorting Genetic Algorithm II (NSGA-II) with the Multi-Attributive Reference Point-Based Approach (MARCOS) for the design of a two-stage helical gearbox incorporating a split slow stage. The optimization aims to simultaneously minimize the gearbox footprint, defined as the base area of the housing, and maximize transmission efficiency, which are inherently conflicting objectives in practical engineering design.

A comprehensive parametric model is developed to represent the geometric configuration, operational constraints, and performance characteristics of the gearbox system. The NSGA-II algorithm is employed to obtain a diverse set of Pareto-optimal solutions over a wide range of transmission ratios. Subsequently, the MARCOS method is applied as a decision-making tool to rank these solutions and determine the most suitable design based on their relative closeness to the ideal alternative.

The results indicate that the proposed framework effectively identifies optimal gearbox configurations that achieve a favorable balance between compactness and efficiency. In particular, the approach proves highly effective for applications with limited installation space and stringent performance requirements. Therefore, the integration of NSGA-II and MARCOS provides a robust and practical decision-support methodology for the optimal design of advanced gear transmission systems.

Keywords: *Two-stage helical gearbox; Split slow stage; NSGA-II; MARCOS method; Multi-objective optimization; Gearbox footprint; Transmission efficiency.*

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TÓM TẮT

Bài báo này trình bày một khung tối ưu hóa đa mục tiêu lai kết hợp thuật toán di truyền phân loại không trội II (NSGA-II) với phương pháp dựa trên điểm tham chiếu đa thuộc tính (MARCOS) để thiết kế hộp số bánh răng cấp chậm phân đôi. Mục tiêu tối ưu hóa là đồng thời giảm thiểu diện tích đáy của hộp số, và tối đa hóa hiệu suất truyền động, đây là những mục tiêu mâu thuẫn nhau trong thiết kế kỹ thuật thực tế.

Một mô hình tham số toàn diện được phát triển để biểu diễn cấu trúc hình học, các ràng buộc vận hành và đặc tính hiệu suất của hộp số. Thuật toán NSGA-II được sử dụng nhằm xác định một tập hợp các giải pháp tối ưu Pareto trên một phạm vi rộng của các tỷ số truyền. Sau đó, phương pháp MARCOS được áp dụng như một công cụ ra quyết định để xếp hạng các giải pháp này và xác định thiết kế phù hợp nhất dựa trên độ gần tương đối của chúng với phương án lý tưởng.

Kết quả cho thấy khung đề xuất xác định hiệu quả các cấu hình hộp số tối ưu đạt được sự cân bằng thuận lợi giữa tính nhỏ gọn và hiệu suất. Đặc biệt, phương pháp này tỏ ra rất hiệu quả đối với các ứng dụng có không gian lắp đặt hạn chế và yêu cầu hiệu suất nghiêm ngặt. Do đó, sự kết hợp giữa NSGA-II và MARCOS cung cấp một phương pháp hỗ trợ quyết định mạnh mẽ và thiết thực cho việc thiết kế tối ưu các hệ thống truyền động bánh răng tiên tiến.

Từ khóa: *Hộp giảm tốc bánh răng trụ hai cấp; Cấp chậm phân đôi; Thuật toán NSGA-II; Phương pháp MARCOS; Tối ưu hóa đa mục tiêu; Diện tích đáy hộp giảm tốc; Hiệu suất truyền động.*

1. INTRODUCTION

Helical gearboxes are widely used in industrial power transmission systems due to their smooth meshing, high load-carrying capacity, and efficiency. Increasing demands for compact and energy-efficient designs have driven the adoption of multi-objective optimization (MOO) approaches to address conflicting criteria such as gearbox size and performance. In particular, minimizing the gearbox footprint (base area) while maximizing transmission efficiency remains a key engineering challenge in space-constrained applications.

Evolutionary algorithms have been extensively applied to gear system design. Yao [1] demonstrated the effectiveness of integrating NSGA-II with decision-making for spur gear optimization under conflicting objectives. Konak et al. [2] highlighted the applicability of multi-objective genetic algorithms in engineering design, while Patil et al. [3] incorporated tribological effects to improve reliability and efficiency. Ananthapadmanabhan et al. [4] further showed that combining global search with selection strategies enhances solution quality.

Recent studies have focused on two-stage helical gearboxes, often incorporating structural modifications such as split stages to improve load distribution and spatial efficiency. Thanh et al. [5] optimized both efficiency and base area, confirming the inherent trade-off between these objectives. Among available techniques, NSGA-II [6] remains a standard approach for generating well-distributed Pareto solutions. In addition, theoretical models such as those by Wang and Wang [7] provide a basis for determining gear geometry under stress constraints.

The decision-making phase is equally important for selecting a preferred solution from the Pareto set. Earlier approaches employed methods such as Taguchi–GRA [8] and genetic algorithms [9], whereas recent studies have adopted structured MCDM techniques. Daoudi et al. [10] applied genetic-based optimization to epicyclic gear systems, while Dinh et al. [11] demonstrated that the MARCOS method provides consistent ranking of non-dominated solutions.

Further research has expanded optimization objectives to include spatial efficiency [12], planetary gear systems [13], vibration and noise reduction [14], gear shifting strategies [15], and component-level optimization such as planet carriers [16].

However, limited studies have addressed hybrid NSGA-II and MARCOS frameworks for two-stage helical gearboxes with a split slow stage. Although this configuration offers advantages in torque distribution and modularity, it also introduces additional complexity in geometry and power losses. Moreover, most existing works treat size and efficiency separately rather than within an integrated decision-support framework.

To overcome these limitations, this study proposes a hybrid multi-objective optimization approach combining NSGA-II and MARCOS to simultaneously minimize gearbox footprint and maximize efficiency. The focus is on two-stage helical gearboxes with a split slow stage, aiming to achieve an optimal balance between compactness and performance for practical engineering applications.

2. OPTIMIZATION PROBLEM

2.1. Determination of gearbox footprint

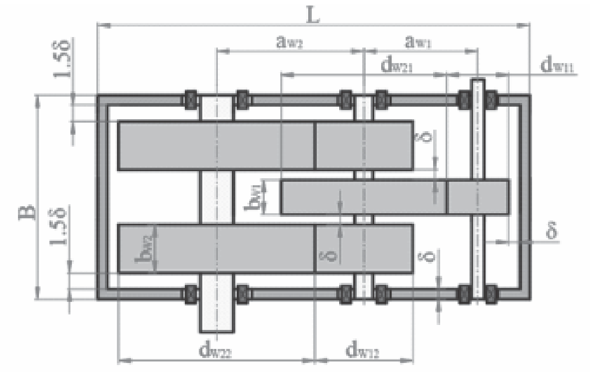

Figure 1. Schema for determining gearbox footprint

The gearbox footprint A_b can be calculated by (Figure 1):

$$A_b = (L \cdot B) \quad (1)$$

Where L and B are computed by (Figure 1):

$$L = d_{w11} + d_{w21}/2 + d_{w12}/2 + d_{w22} + 4 \cdot \delta \quad (2)$$

$$B = b_{w1} + 2 \cdot b_{w2} + 7 \cdot \delta \quad (3)$$

In which, δ = 7÷10 (mm) [17]; b_{wi}, d_{w1i}, d_{w2i} is the gear width, the pitch diameter of the pinion and the gear of ith stage (i=1÷2), respectively. These parameters can be determined by:

$$b_{wi} = X_{bai} \cdot a_{wi} \quad (4)$$

$$d_{w1i} = 2 \cdot a_{wi}/(u_i + 1) \quad (5)$$

$$d_{w2i} = 2 \cdot a_{wi} \cdot u_i \cdot/(u_i + 1) \quad (6)$$ ☞

Where X_{bai} and a_{wi} (i=1÷2) denote the face width coefficient and the center distance of the i^{th} stage. a_{wi} can be found by [17]:

$$a_{wi} = k_a \cdot (u_i + 1) \cdot \sqrt[3]{T_{1i} \cdot k_{H\beta}/([AS_i]^2 \cdot u_i \cdot X_{bai})} \quad (7)$$

In which, T_{1i} (i=1÷2) is the torque on the pinion of i^{th} stage which can be computed by:

$$T_{11} = T_{out}/\left(u_{gb} \cdot \eta_{hg}^2 \cdot \eta_b^3\right) \quad (8)$$

$$T_{12} = T_{out}/\left(2 \cdot u_2 \cdot \eta_{hg} \cdot \eta_{be}^2\right) \quad (9)$$

2.2. Calculation of gearbox efficiency

The gearbox efficiency (%) is calculated by:

$$\eta_{gb} = 100 - \frac{100 \cdot P_l}{P_{in}} \quad (10)$$

Where P_l is the total power loss in the gearbox which is determined by [18]:

$$P_l = P_{lg} + P_{lb} + P_{ls} + P_{Z0} \quad (11)$$

In (11), P_{lg}, P_{lb}, P_{ls}, and P_{Z0} denote the power losses due to gear meshing, bearing friction, seal drag, and idle motion, respectively. These components are calculated following the procedure described in [19].

2.3. Objective functions and constraints

The optimization problem is formulated as a bi-objective minimization – maximization model as follows:

- Decision variables:

$$X = \{u_1, X_{ba1}, X_{ba2}\} \quad (12)$$

Where u_1 is the gear ratio of stage 1; X_{ba1}, X_{ba2} are the face width coefficients of stage 1 and 2.

- Objective functions:

$$f_1(X) = A_b \rightarrow min \quad (13)$$

$$f_2(X) = \eta_{gb} \rightarrow max \quad (14)$$

Constraints:

- Gear ratio bounds [17]:

$$1 \le u_i \le 9 \quad (15)$$

- Face width coefficient bounds [17]:

$$0.25 \le X_{bai} \le 0.4 \quad (16)$$

3. METHODOLOGY

3.1. NSGA-II for multi-objective optimization

NSGA-II, proposed by Deb et al. [6], is one of the most widely used evolutionary algorithms for multi-objective optimization problems. In this study, it is employed to generate a set of Pareto-optimal solutions that simultaneously minimize the gearbox base area and maximize transmission efficiency.

Each individual in the population represents a candidate gearbox design, which is encoded by the following decision vector:

$$X = \{u_1, X_{ba1}, X_{ba2}\} \quad (17)$$

The key features of NSGA-II utilized in this study include fast non-dominated sorting, an elitist selection mechanism, and the preservation of solution diversity through crowding distance. The algorithm was executed for multiple values of the total transmission ratio u_h, and for each case, a corresponding set of Pareto-optimal solutions was obtained.

The NSGA-II algorithm was implemented using the following parameter settings: a population size of 100 individuals and a maximum of 200 generations per optimization run. The crossover probability was set to 0.9 and the mutation probability to 0.1. The distribution index for SBX crossover (η_s) and polynomial mutation (η_m) were both set to 20. These values were selected based on standard practices in evolutionary multi-objective optimization [6], and were further validated through experimentation to ensure convergence and diversity of the Pareto-optimal front in the context of split-stage helical gearbox design.

3.2. Methodology for solving MCDM

To identify the most suitable compromise solution from the Pareto front, the MARCOS method was employed. This approach ranks candidate solutions based on their relative closeness to the ideal (AI) and anti-ideal (AAI) alternatives through a normalized utility-based framework. The application of the MARCOS technique follows a sequence of systematic steps, which should be implemented in accordance with the procedures described in [20]:

Step 1: Create the initial decision-making matrix:

$$X = \begin{bmatrix} x_{11} & \cdots & x_{1n} \\ x_{21} & \cdots & x_{2n} \\ \vdots & \cdots & \vdots \\ x_{mn} & \cdots & x_{mn} \end{bmatrix} \tag{18}$$

Where m is the number of alternatives and n is the number of criteria.

Step 2: Extend the matrix to include ideal (AI) and anti-ideal (AAI) solutions:

$$X = \begin{array}{c} AAI \\ A_1 \\ A_2 \\ \vdots \\ A_m \\ AI \end{array} \begin{bmatrix} x_{aa1} & \cdots & x_{aan} \\ x_{11} & \cdots & x_{1n} \\ x_{21} & \cdots & x_{2n} \\ \vdots & \vdots & \vdots \\ x_{m1} & \cdots & x_{mn} \\ x_{ai1} & \cdots & x_{ain} \end{bmatrix} \tag{19}$$

In which, AAI = min(x_{ij}) and AI = max(x_{ij}) for efficiency (to maximize); AAI = max(x_{ij}) and AI = min(x_{ij}) for bottom area (to minimize); i = 1, 2,..., m; j = 1, 2,..., n.

Step 3: Normalize the extended matrix to obtain u_{ij}:

$$u_{ij} = x_{AI} / x_{ij} \tag{20}$$

$$u_{ij} = x_{ij} / x_{AI} \tag{21}$$

Wherein (20) is applied for bottom area, and (20) is applied for efficiency.

Step 4: Compute the weighted normalized matrix C = $[c_{ij}]_{m\times n}$ by:

$$c_{ij} = u_{ij} \cdot w_j \tag{22}$$

Where w_j is the weight coefficient of criterion j.

Step 5: Calculate the utility of alternatives K_i^- and K_i^+ by:

$$K_i^- = S_i / S_{AAI} \tag{23}$$

$$K_i^+ = S_i / S_{AI} \tag{24}$$

In (22) and (23), S_i is found by:

$$S_i = \sum_{i=1}^{m} c_{ij} \tag{25}$$

Step 6: Calculate the overall utility function f(K_i) for each alternative by:

$$f(K_i) = \frac{K_i^+ + K_i^-}{1 + \frac{1 - f(K_i^+)}{f(K_i^+)} + \frac{1 - f(K_i^-)}{f(K_i^-)}} \tag{26}$$

with:

$$f(K_i^-) = K_i^+ / (K_i^+ + K_i^i) \tag{27}$$

$$f(K_i^+) = K_i^- / (K_i^+ + K_i^i) \tag{28}$$

Step 7: Rank the alternative's order by maximizing $f(K_i)$.

4. RESULTS AND DISCUSSIONS

4.1. Trends of gearbox bottom area and efficiency with respect u_h

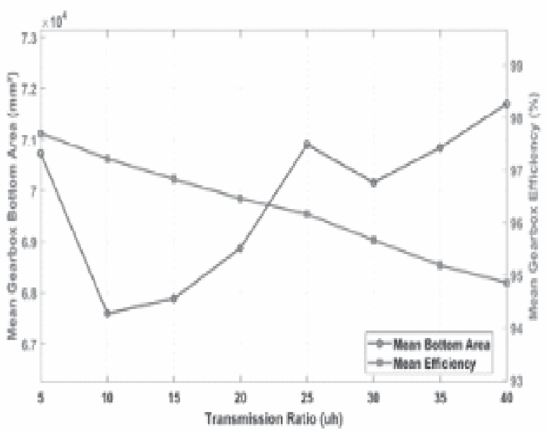

Figure 2. Mean trend of gearbox bottom area and efficiency vs. u_h

Figure 2 presents the variation of the mean gearbox bottom area and mean transmission efficiency as functions of the overall transmission ratio u_h. It can be observed that the efficiency gradually decreases with increasing u_h, from approximately 97.9% at u_h = 5 to about 94.5% at u_h = 40. In contrast, the bottom area exhibits a non-monotonic behavior, initially decreasing and then increasing once u_h exceeds 20. This trend reflects the inherent trade-off in gearbox design: higher transmission ratios typically require larger gear diameters or increased face widths, which lead to a greater footprint and higher cumulative mechanical losses, particularly in configurations with a split stage.

4.2. Pareto front distribution by transmission ratio

The Pareto fronts generated by NSGA-II for different values of u_h are illustrated in Figure 3, where each front represents the trade-off between gearbox bottom area and transmission efficiency. It can be observed that as u_h increases, the Pareto fronts shift toward the right and downward, indicating a simultaneous degradation in both objectives. For lower transmission ratios (e.g., u_h = 5–15), the solution space is relatively wide, providing multiple design options with high efficiency and compact geometry. In contrast, at higher ratios (e.g., u_h = 35–40), the Pareto fronts become narrower and more concave, reflecting a reduced number of optimal trade-offs and tighter design constraints. These trends confirm that increasing transmission ratios limit both spatial and energetic flexibility, particularly in split-output gearbox configurations.

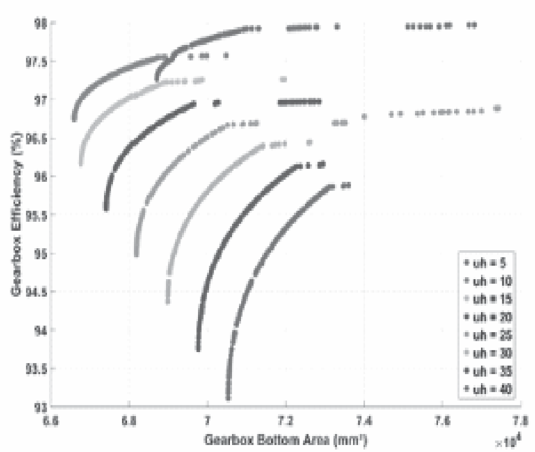

Figure 3. Pareto fronts for various u_h

4.3. Regression analysis of u_1 vs. u_h

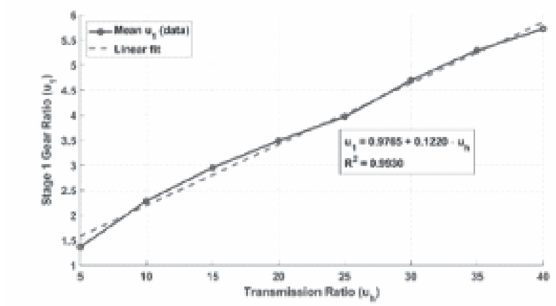

Figure 4. Linear fit of mean u_1 vs. u_h

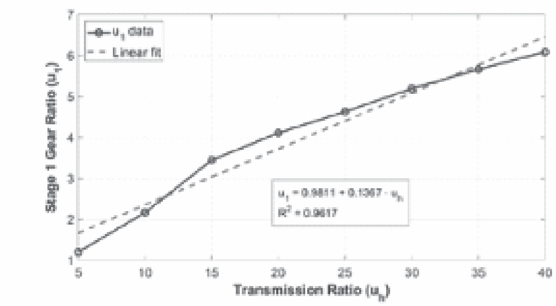

Figure 5. Linear fit of MARCOS-based u_1 vs. u_h

To support preliminary design decisions, regression models were constructed to express the stage 1 gear ratio u_1 as a function of the total ratio u_h. Two cases were analyzed:

- Mean-based trend (Figure 4) ($R^2 = 0.9930$):

$$u_1 = 0.1220 \cdot u_h + 0.9765 \tag{29}$$

- MARCOS-based optimal values (Figure 5) ($R^2 = 0.9617$):

$$u_1 = 0.1367 \cdot u_h + 0.9811 \tag{30}$$

4.4. MARCOS-based optimal solutions

Table 1 summarizes the optimal solutions selected from the Pareto fronts using the MARCOS method for each value of u_h. All face width coefficients were fixed at 0.25 to emphasize the effect of u_1 on performance.

Table 1. Optimal Solutions Selected by MARCOS for Each u_h

u_h	u_1	X_{ba1}	X_{ba2}	Area (mm²)	Efficiency (%)
5	1.20	0.25	0.25	70,966.82	97.91
10	2.16	0.25	0.25	67,600.91	97.33
15	3.44	0.25	0.25	66,867.07	96.44
20	4.11	0.25	0.25	67,561.60	96.01
25	4.62	0.25	0.25	68,447.02	95.66
30	5.19	0.25	0.25	69,286.79	95.21
35	5.66	0.25	0.25	70,157.68	94.85
40	6.08	0.25	0.25	71,023.34	94.51

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The results clearly show a decreasing efficiency trend and an increasing bottom area, aligned with the trade-offs observed earlier. Notably, the selected u_1 values follow a near-linear progression and match well with the regression model in Figure 4.

This confirms that MARCOS not only identifies high-quality compromise solutions but also does so consistently across a wide transmission range, preserving both engineering feasibility and spatial constraints.

5. CONCLUSIONS

This study proposed a hybrid optimization framework that integrates the NSGA-II evolutionary algorithm with the MARCOS multi-criteria decision-making method for the design of a two-stage helical gearbox with a split slow stage. The optimization simultaneously targets two key objectives: minimizing the gearbox base area and maximizing transmission efficiency, which are critical in space-constrained and high-performance applications.

The NSGA-II algorithm successfully generated well-distributed Pareto-optimal solutions over a wide range of transmission ratios (u_h), clearly revealing the trade-off between geometric compactness and efficiency. The observed shifts in Pareto fronts and associated trend analyses indicate that increasing transmission ratios progressively restrict the feasibility design space and reduce achievable performance, particularly due to the added structural complexity of the split-output configuration.

The MARCOS method was then effectively employed to identify the most suitable solutions from each Pareto front. The selected designs exhibited consistent and nearly linear trends in the allocation of gear ratios (u_1), while maintaining a balanced performance between the objectives. Furthermore, regression analysis enabled the derivation of empirical design guidelines, supporting efficient decision-making in the preliminary design stage.

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