VIBROACOUSTIC BEHAVIOR OF A CLAMPED DOUBLE-COMPOSITE PLATE FILLED WITH AN AIR CAVITY IN THERMAL ENVIRONMENT

ỨNG XỬ DAO ĐỘNG ÂM QUA TẤM KÉP COMPOSITE CÓ KHOANG KHÍ NGÀM BỐN CẠNH TRONG MÔI TRƯỜNG NHIỆT

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ABSTRACT

This paper is a study of the vibroacoustic behavior of a finite laminated double - composite rectangular plate filled with an air cavity under a sound wave excitation in thermal envi-ronments is investigated analytically. Using the method of modal decomposition, a double Fou-rier series solution is obtained to characterize the vibroacoustic performance of the structure. The sound transmission loss (STL) is calculated from the ratio of incident to transmitted acous-tic powers.

Keywords: *Vibroacoustic behavior; Laminated composite plate; Sound transmission loss; Thermal loads.*

TÓM TẮT

Bài báo này là một nghiên cứu về ứng xử dao động âm của một tấm kép composite lớp hình chữ nhật, hữu hạn chứa khoang khí bị kích thích bởi sóng âm trong môi trường nhiệt được nghiên cứu và phân tích. Sử dụng phương pháp tách mốt, chuỗi Fourier kép để mô tả dạng dao động âm của kết cấu tấm. Tổn thất truyền âm (STL) được tính theo tỷ số công suất âm truyền tới với công suất âm truyền qua.

Từ khóa: *Ứng xử dao động âm; Tấm composite lớp; Tổn thất truyền âm; Tải nhiệt.*

1. INTRODUCTION

The double-plate is widely used as in construction structures, ships, turbofan, aerospace, etc. These structures are invariably exposed to the thermal and noise environment in their ser-vice life, especially as a component of the hypersonic aircraft. Therefore, research on sound insulation capacity of double-plate in thermal environment has received much attention from researchers in the world to produce the most optimal structures.

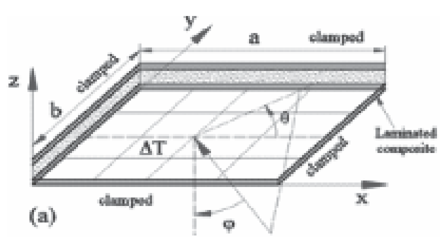
The vibroacoustic behavior of the double-plates filled with absorbent materials has been extensively investigated. For example, Carneal and Fuller [1] presented an analytical and experimental study of active structural acoustic ☞

control of noise transmission through aluminum double panel systems. Chazot and Guyader [2] studied the sound transmission loss through finite double-panels by using a patch-mobility method. This method and FEM give the same results when comparing transmission loss of double aluminum plates. In [3] a finite element formulation for sound transmission through double wall sandwich panels with viscoelastic core is presented by Larbi et al. Lu and Xin [5] presented an analytical approach to investigate the sound transmission across rectangular metallic (isotropic) double-panel structure containing an air cavity under various boundary constraints by using the method of modal function and the weighted residual (Galerkin) method. More recently, Thinh and Thanh [6-9] calculated the sound transmission loss through a finite single and double-composite plate in thermal and air environment with different boundary conditions (clamped and simply surpported). The effect of some parameters on the sound insulation ability of the plate structure is considered.

The objective of this study is to develop a model to describe accurately the sound transmission across a finite clamped laminated double-composite plate filled with air cavity in thermal environment. Thereby, assessing the influence of several key system parameters on sound transmission loss including the faceplate thickness and the air cavity thickness are systematically examined.

2. THEORETICAL FORMULATION

2.1. Geometry and assumptions



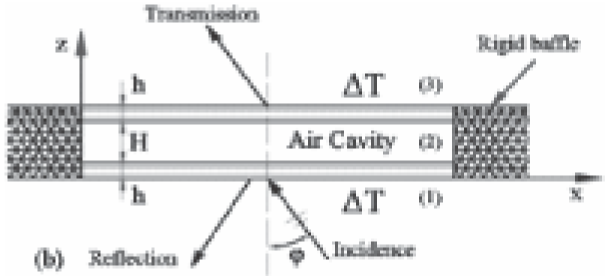


Figure 1. Schematic of STL through a double-composite plate: (a) Global view and (b) side view.

Supposedly, a finite rectangular double-composite plate with air cavity is in an infinite large acoustic rigid baffle. The two single-plates are orthotropic laminated composite and have similar geometric parameters and mechanical properties. The bottom and upper plates (Fig. 1) have thicknesses h and is separated by an air cavity of thickness H. The double-plate partition divides the spatial space into two fields, i.e., sound incidence field ($z < 0$) and sound transmitting field ($z > H + 2h$).

A plane sound wave varying harmonically in time is oblique (with the incident angle, φ and azimuth angle, θ) and stimulates the vibration of the bottom plate. This vibration changes the pressure in the air cavity and causes vibration of the upper plate then the sound wave is transmitted into the upper domain.

2.2. Laminated composite plate dynamics

The vibroacoustic behavior of an orthotropic symmetric double- composite plate with air cavity in thermal environment (Fig. 1) induced by sound excitation is governed by [8, 9]:

$$D_{11}\frac{\partial^4 w_1(x,y;t)}{\partial x^4}+2(D_{12}+2D_{66})\frac{\partial^4 w_1(x,y;t)}{\partial x^2\partial y^2}+D_{22}\frac{\partial^4 w_1(x,y;t)}{\partial y^4}+m^*\frac{\partial^2 w_1(x,y;t)}{\partial^2 t}-\left(N_1\frac{\partial^2 \mathrm{w}(x,y;t)}{\partial x^2}+N_2\frac{\partial^2 \mathrm{w}(x,y;t)}{\partial y^2}\right)=j\omega\rho_0\left[\Phi_1(x,y,z;t)-\Phi_2(x,y,z;t)\right] \quad (1)$$

$$D_{11}\frac{\partial^4 w_1(x,y;t)}{\partial x^4}+2(D_{12}+2D_{66})\frac{\partial^4 w_1(x,y;t)}{\partial x^2\partial y^2}+D_{22}\frac{\partial^4 w_1(x,y;t)}{\partial y^4}+m^*\frac{\partial^2 w_1(x,y;t)}{\partial^2 t}-\left(N_1\frac{\partial^2 \mathrm{w}(x,y;t)}{\partial x^2}+N_2\frac{\partial^2 \mathrm{w}(x,y;t)}{\partial y^2}\right)=j\omega\rho_0\left[\Phi_2(x,y,z;t)-\Phi_3(x,y,z;t)\right] \quad (2)$$

Where $w_{1,2}$ are the transverse displacements of the two single-plates; D_{ij} (ij = 11,12,22,66) are the flexural rigidities; m* is the surface density of the upper and bottom plates; ρ_0 is the air density, ω is the angular frequency of the incident sound and Φ_i (i = 1,2,3) denote the velocity potentials for the acoustic fields in the proximity of the two plates, corresponding to the sound incidence, the air cavity, and the transmitting field, respectively. N_i (i = 1,2) are constants determined by:

$$N_i=\sum_{k=1}^{N}\int_{z_{k-1}}^{z_k}[Q_{ij}]_k\alpha_j\Delta T dz \qquad (i,\ j\ =\ 1,2) \quad (3)$$

Where: α_j (j = 1,2) are coefficients of thermal expansion in longitudinal and transverse direc-tion; ΔT is temperature difference (is assumed to be constant in this study).

The displacements of the two composite faceplates induced by the incident sound can be expressed as:

$$w_1(x,y;t)=w_0\,.e^{-j(k_x x+k_y y-\omega t)};\ w_2(x,y;t)=w_0\,.e^{-j(k_x x+k_y y-\omega t)} \quad (4)$$

The acoustic velocity potential in the incidence field (Fig.1) is defined:

$$\Phi_1(x,y,z;t)=I.e^{-j(k_x x+k_y y+k_z z-\omega t)}+\beta.e^{-j(k_x x+k_y y-k_z z-\omega t)} \quad (5)$$

Where I and β are the amplitudes of the incident (positive-going) and the reflected plus radiat-ed (negative-going) waves, respectively. Similarly, the velocity potential in the air cavity can be written as:

$$\Phi_2(x,y,z;t)=\varepsilon.e^{-j(k_x x+k_y y+k_z z-\omega t)}+\psi.e^{-j(k_x x+k_y y-k_z z-\omega t)} \quad (6)$$

Where ε is the amplitude of the positive-going wave and ψ is the amplitude of the negative-going wave. In the transmitting field adjacent to the transmitting upper plate, there exist no reflected waves, and therefore the velocity potential in the transmitted waves is given by:

$$\Phi_3(x,y,z;t)=\gamma.e^{-j(k_x x+k_y y+k_z z-\omega t)} \quad (7)$$

Where γ is the amplitude of the transmitted (positive-going) wave.

These wavenumbers are determined by the incident angle, φ and azimuth angle, θ of the incident sound wave as:

$$k_x=k_0\sin\varphi\cos\theta;k_y=k_0\sin\varphi\sin\theta;k_z=k_0\cos\varphi \quad (8)$$

Where $k_0 = \omega/c_0$ is the acoustic wave number in air and c_0 is the acoustic speed in the air.

The boundary conditions for the clamped double-composite plate can be expressed as: ☞

$$At\ x=0,\ a,\quad w_1=w_2=0,\ \frac{\partial w_1}{\partial x}=\frac{\partial w_2}{\partial x}=0\ ;$$
$$\text{At y}=0,\ \text{b},\quad w_1=w_2=0,\ \frac{\partial w_1}{\partial y}=\frac{\partial w_2}{\partial y}=0 \tag{9}$$

At the air-plate interface, the normal velocity is continuous, yielding the corresponding velocity compatibility condition equations:

$$At\ z=0,\ -\frac{\partial \Phi_1}{\partial z}=j\omega w_1;\ z=\text{h},-\frac{\partial \Phi_2}{\partial z}=j\omega w_1;$$
$$z=h+H,\ -\frac{\partial \Phi_2}{\partial z}=j\omega w_2;\ z=2\text{h}+\text{H},-\frac{\partial \Phi_3}{\partial z}=j\omega w_2 \tag{10}$$

The flexural motions of the bottom and upper plates induced by a time-harmonic incident plate sound wave can be expressed in the form of double series as:

$$w_1(x,y;t)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\varphi_{mn}(x,y)q_{1,mn}(t);$$
$$w_2(x,y;t)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\varphi_{mn}(x,y)q_{2,mn}(t) \tag{11}$$

Where the modal function φ_{mn} and modal displacements $q_{i,mn}$ (i = 1,2) for the clamped bound-ary conditions are given by [7, 8, 9]:

$$\varphi_{mn}(x,y)=\left(1-\cos\frac{2m\pi x}{a}\right)\left(1-\cos\frac{2n\pi x}{b}\right) \tag{12}$$

$$q_{1,mn}(t)=\alpha_{1,mn}e^{j\omega t},\quad q_{2,mn}(t)=\alpha_{2,mn}e^{j\omega t} \tag{13}$$

Where $\alpha_{1,mn}$; $\alpha_{2,mn}$ are the modal coefficients of the bottom and supper plates, respectively.

Thus, the velocity potentials for the sound incident region, air cavity, and sound transmit-ting region may be expressed as:

$$\Phi_1(x,y,z;t)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}I_{mn}\varphi_{mn}e^{-j(k_z z-\omega t)}+\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\beta_{mn}\varphi_{mn}e^{-j(-k_z z-\omega t)} \tag{14}$$

$$\Phi_2(x,y,z;t)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\varepsilon_{mn}\varphi_{mn}e^{-j(k_z z-\omega t)}+\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\psi_{mn}\varphi_{mn}e^{-j(-k_z z-\omega t)} \tag{15}$$

$$\Phi_3(x,y,z;t)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\gamma_{mn}\varphi_{mn}e^{-j(k_z z-\omega t)} \tag{16}$$

Where the coefficients I_{mn}, β_{mn}, ε_{mn}, ψ_{mn} and γ_{mn} for the clamped double-composite plate are determined by:

$$\chi_{mn}=\frac{4}{ab}\int_0^b\int_0^a \chi e^{-j(k_x x+k_y y)}\cos\frac{2m\pi x}{a}\cos\frac{2n\pi x}{b}dxdy \tag{18}$$

Here, the symbol χ can be referred to any of the coefficients I_{mn}, β_{mn}, ε_{mn}, ψ_{mn} and γ_{mn}.

Using the displacement continuity condition at air-plate interfaces [10] and substituting Eqs. (14) - (16) into Eqs. (18) leads to:

$$\beta_{mn}=I_{mn}-\frac{\omega}{k_z}\alpha_{1,mn}\ ;\ \varepsilon_{mn}=\frac{\omega\left(\alpha_{1,mn}e^{jk_z H}-\alpha_{2,mn}\right)}{k_z\left(e^{-jk_z(h-H)}-e^{-jk_z(H+h)}\right)} \tag{19}$$

$$\psi_{mn}=\frac{\omega\left(\alpha_{2,mn}-\alpha_{1,mn}e^{-jk_z H}\right)}{k_z\left(e^{jk_z(h-H)}-e^{jk_z(H+h)}\right)};\ \gamma_{mn}=\frac{\omega\alpha_{2,mn}e^{jk_z(H+2h)}}{k_z} \tag{20}$$

Substituting Eqs. (11) and (19) - (20) into Eqs. (1) - (2) and applying the orthogonal properties of modal functions, one gets:

$$\ddot{q}_{1,mn}(t)+\omega_{1,mn}^2 q_{1,mn}(t)-\frac{j\omega\rho_0}{m^*}\begin{bmatrix}(I_{mn}-\varepsilon_{mn})e^{-j(k_z z-\omega t)}+(\beta_{mn}-\psi_{mn})e^{-j(-k_z z-\omega t)}\\ +4N_1\left(\frac{m\pi}{a}\right)^2+4N_2\left(\frac{n\pi}{b}\right)^2\end{bmatrix}=0 \tag{21}$$

$$\ddot{q}_{2,mn}(t)+\omega_{2,mn}^2 q_{2,mn}(t)-\frac{j\omega\rho_0}{m^*}\begin{bmatrix}(\varepsilon_{mn}-\gamma_{mn})e^{-j(k_z z-\omega t)}+\psi_{mn}e^{-j(-k_z z-\omega t)}\\ +4N_1\left(\frac{m\pi}{a}\right)^2+4N_2\left(\frac{n\pi}{b}\right)^2\end{bmatrix}=0 \tag{22}$$

Where: $\ddot{q}_{i,mn}(t)\ (i=1,2)$ are the second derivatives of the displacement of the bottom and upper plates, respectively, $q_{i,mn}(t)$;

$\omega_{i,mn}$ (i = 1,2) are the natural frequencies of the two single-leaf plates are orthotropic laminated composite are determined by:

$$\omega_{i,mn}^2 = \frac{\pi^4}{m^* b^4}\left[D_{11} m^4 \frac{48}{9}\left(\frac{b}{a}\right)^4 + 2\left(D_{12} + 2D_{66}\right)\frac{16}{9} m^2 n^2 \left(\frac{b}{a}\right)^2 + \frac{48}{9} D_{22} n^4\right] + \frac{\pi^2}{I^*}\left[N_1\left(\frac{m}{a}\right)^2 + N_2\left(\frac{n}{b}\right)^2\right](i=1,2) \quad (23)$$

Where: $I^* = \sum_{k=1}^{n} \rho_0^{(k)}\left(h_{k+1} - h_k\right)$. Therefore, the modal coefficients, $\alpha_{i,mn}$ is determined by used Eq. (13), Eqs. (21) and (22) can be rewritten in matrix form as:

$$\begin{bmatrix} X_{11} & X_{12} \\ X_{21} & X_{22} \end{bmatrix}\begin{Bmatrix} \alpha_{1,mn} \\ \alpha_{2,mn} \end{Bmatrix} = \begin{Bmatrix} Y \\ 0 \end{Bmatrix} \quad (24)$$

With:

$$\begin{aligned}
X_{11} &= \omega_{1,mn}^2 - \omega^2 - \frac{j\omega^2\rho_0}{k_z m^*}\left[\frac{e^{jk_z H}}{e^{-jk_z(h-H)} - e^{-jk_z(H+h)}} - \frac{e^{-jk_z H}}{e^{jk_z(h-H)} - e^{jk_z(H+h)}}\right] \\
X_{12} &= \frac{j\omega^2\rho_0}{k_z m^*}\left[\frac{1}{e^{-jk_z(h-H)} - e^{-jk_z(H+h)}} - \frac{1}{e^{jk_z(h-H)} - e^{jk_z(H+h)}}\right] \\
X_{21} &= \frac{j\omega^2\rho_0}{k_z m^*}\left[\frac{e^{jk_z H}}{e^{-jk_z(h-H)} - e^{-jk_z(H+h)}} - \frac{e^{-jk_z H}}{e^{jk_z(h-H)} - e^{jk_z(H+h)}}\right] \\
X_{22} &= \omega_{2,mn}^2 - \omega^2 - \frac{j\omega^2\rho_0}{k_z m^*}\left[\frac{1}{e^{jk_z(h-H)} - e^{jk_z(H+h)}} - \frac{1}{e^{-jk_z(h-H)} - e^{-jk_z(H+h)}} + e^{jk_z(H+2h)}\right] \\
Y &= \frac{2j\omega\rho_0 I_{mn}}{m^*}
\end{aligned} \quad (25)$$

After solving the system of equations (24), the unknown coefficients $\alpha_{1,mn}$ and $\alpha_{2,mn}$ are determined, all the other quantities such as $w_{1,mn}$, $w_{2,mn}$ and the coefficients (β_{mn}, ε_{mn}, ψ_{mn} and γ_{mn}) are also determined. Thence, the analysis of sound transmission across a clamped double-composite plate is completely solved.

2.3. Definition of sound transmission loss

The power of incident sound is defined [7, 8]:

$$\Pi_1 = \frac{1}{2}\mathrm{Re}\iint_A p_1 v_1^* dA \quad (26)$$

The transmitted sound power is defined [7, 8]:

$$\Pi_3 = \frac{1}{2}\mathrm{Re}\iint_A p_3 v_3^* dA \quad (27)$$

Where: v_1^*, v_3^*, p_1, p_3 are the local acoustic velocity, the sound pressure in the incident field and the transmitted field, respectively.

The sound transmission loss across the laminated double-composite plate is defined by [7, 8, 9]:

$$STL = 10\log_{10}\left(\frac{\Pi_1}{\Pi_3}\right) \quad (28)$$

3. NUMERICAL RESULTS AND DISCUSSION

3.1. Validation

The present analytical approach is validated by comparing the predicted STL with the theoretical and experimental results of Carneal and Fuller [1] as shown in Fig. 2 for a finite double-plate is excited by wave varying harmonically with incident angle, $\varphi = 30°$ and azimuth angle, $\theta = 30°$ and thermal load, $\Delta T = 0°C$. The double-plate partition considered consists of two identical metallic (isotropic plates). The geometrical dimensions of the plates are: length of plate a = 1m, width of plate b = 1m. The plate have thickness h = 2mm while the thickness of the air cavity is H = 0.08m. The mechanical properties of aluminum materials are: E = 70GPa; $\rho = 2700 kg/m^3$; $\nu = 0.33$. Speed of sound in air, c = 343 m/s; the density of the air, $\rho_0 = 1.21\ kg/m^3$ and the initial amplitude, $I_0 = 1\ m^2/s$. ☞

Fig. 2 indicates clearly that the present analytical predictions are in good agreement with the experimental measurements than the theoretical predictions of Carneal and Fuller [1]. Notice first that the experimental results are not reliable for frequencies below 50 Hz where the flanking paths of the test facility play a prominent role in measurements. Second, the extra dips in the experiment curve compared with the theoretical curves associated with 240-270 Hz and 490-570 Hz modes are attributed to the imperfect normal acoustic plane wave and/or the structural flanking path, as emphasized by Carneal and Fuller [1].

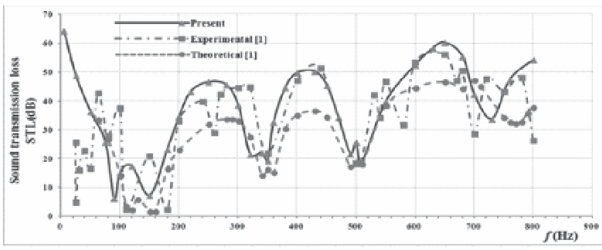


Figure 2. Comparison of the present analytical predictions with the theoretical and experi-mental results of Carneal and Fuller [1].

3.2. Influence of faceplate thickness (h) on STL

The influence of composite materials on STL through a finite clamped double-Graph-ite/Epoxy composite plate is studied in this section by selecting three values of plate thickness are chosen: h = 2mm, 5mm and 10 mm are excited by wave varying harmonically with incident angle, $\varphi = 30°$ and azimuth angle, $\theta = 30°$ under thermal load, $\Delta T = 20°C$. The composite plates are composed of 8 balanced, symmetrical layers, with a configuration of [0/90/0/90]s with the mechanical properties: $E_1 = 137GPa$; $E_2 = 10GPa$; $G_{12} = 5$ GPa; $\upsilon = 0.3$; $\rho = 1590$ kg/m^3. The geometrical dimensions of the plates are: length of plate, a = 1m; width of plate, b = 1m and thickness of the air cavity, H = 0.08 m remain unchanged. Speed of sound in air, c = 343 m/s; the density of the air, $\rho_0 = 1.21$ kg/m^3 and the initial amplitude, $I_0 = 1$ m^2/s.

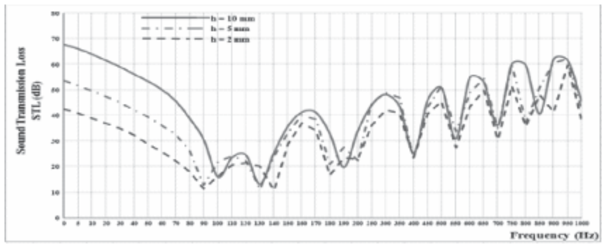


Figure 3. Effects of plate thickness on STL

According to Fig. 3, the STL value increases sharply when increasing the thickness of the bottom and upper plates. The effect of thickness of faceplate for STL is particularly strong at frequencies lower than 100Hz under thermal load. This is a very important region when de-signing finite dual sound insulation plates in practice. In the higher frequency region (f > 100 Hz), the effect of thermal loading is stronger and the strong interaction between the individual behavior of the single plates (top and bottom) and the overall behavior of the finite double plate system makes the peaks and troughs appear denser and more complex.

3.3. Influence of air cavity thickness (H) on STL

To demonstrate the influence of air cavity thickness on STL, the STLs are calculated for a finite double-Graphite/Epoxy composite plate under thermal loads, $\Delta T = 0°C$ and $\Delta T = 20°C$ with selected values of air cavity thickness: H = 0.02m; 0.04m and 0.08 m; $h_1 = h_2 = 2$ mm, as shown in Fig. 4. The other geometrical and material parameters are the same as section 4.2 and normal sound excitation is imposed.

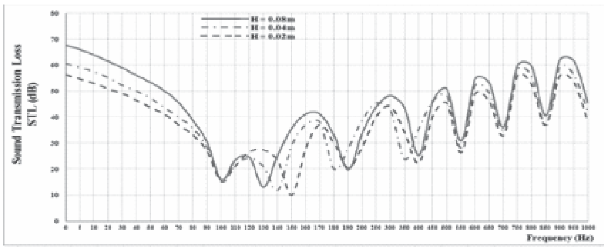


Figure 4. Effects of air cavity thickness on STL

According to Fig. 4, the first dip position does not depend on the air cavity thickness because it completely depends on the surface density of the plate. However, the second dip position changes drastically when the air cavity thickness increases (moving toward the lower frequency) due to the plate-cavity-plate resonance and thermal load a major role in this case. Their remaining positions are almost unchanged because the plate-cavity-plate is operating synchronously under thermal load.

4. CONCLUSIONS

In this investigation, an analytical model was developed to study the sound transmission loss through a finite double-laminated composite plate with the clamped boundary conditions excited by a plane sound wave varying harmonically in thermal enviroment. From the results obtained, some conclusions can be drawn:

• The theoretical predictions are in good agreement with existing results.

• Increasing the plate thickness (equivalent to increasing the surface mass) enhances con-siderably the sound insulation property of the clamped double-plates. The influence of plate thickness on STL is particularly strong for finite systems at low frequencies.

• As the thickness of the air cavity increases, the sound insulation capacity of the double plate also increases but is not as strong as increasing the thickness of bottom and upper plate.❖

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