DYNAMIC SIMULATION AND ANALYSIS OF BICYCLE FRAME USING LAPLACE TRANSFORM AND MATLAB SIMULINK

PHÂN TÍCH VÀ MÔ PHỎNG ĐỘNG LỰC HỌC KHUNG XE ĐẠP SỬ DỤNG PHÉP BIẾN ĐỔI LAPLACE VÀ MATLAB SIMULINK

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ABSTRACT

This article focused to the dynamic analysis of a bicycle frame including the simulation of the acting dynamic forces and the structural analysis of the frame. The dynamic forces from the road acting on the frame are determined from the equation of motion of Multi Degree of Freedom (MDOF) systems with MATLAB and the byke is considered moving on road with sinusoidal profile. The paper presents the following problems: the relationship between sinusoidal road profile and dynamic forces, natural frequencies and dynamic behaviors of the frame. Based on the results, we can determine the dynamic characteristics of the frame to avoid resonance and strengthen weak regions of it.

Keywords: *Bicycle Frame; Laplace transform; Simulation Analysis; Dynamic Analysis.*

TÓM TẮT

Bài báo nói về việc phân tích động lực học của khung xe đạp, bao gồm mô phỏng các lực tác động và phân tích kết cấu khung. Lực từ mặt đường tác động lên khung được xác định từ phương trình chuyển động của hệ thống nhiều bậc tự do (MDOF) dùng MATLAB, và xe đạp được coi là chuyển động trên mặt đường có dạng Sin. Bài báo trình bày các vấn đề sau: mối quan hệ giữa biên dạng mặt đường hình sin và các lực động, tần số riêng và phản ứng động lực học của khung xe. Dựa trên kết quả, chúng ta có thể xác định các đặc tính động lực học của khung để tránh cộng hưởng và gia cố các vùng yếu của khung.

Từ khóa: *Khung xe; Chuyển đổi Laplace; Mô phỏng; Phân tích động lực học.*

1. INTRODUCTION

This article presents the dynamic analysis using MATLAB. We have conducted research on bicycle frame based on differential equation system and MATLAB Simulink and Laplace transform method that are widely used to for modelling and simulation of dynamic systems.

Firstly, the differential equation system are established to describe dynamic motion on a road shaped as basic form as sinus, step and pulse. The shape of road has effect on the frame of bike. This effect is explored using dynamic analysis. This dynamic analysis with the dynamic force of the road surface acting on the vehicle is expected to see the maximum deformation and dynamic force of the road surface acting on the frame happened for mountain bike frame model.

2. REVIEW OF BICYCLE FRAME

The basic requirements for a bicycle frame are top tube, head tube, down tube, chain stays, seat stay, and seat tube. The double triangle shape of bicycle frame is known as a diamond frame. Various loads are acting on the different parts of a bicycle frame. In this study, the rider with 100 kg was considered to analyze the static and dynamic analysis of the alunimium alloy Al6061 T6 frame by FEM.

The material properties are shown in Table 1. The primary dimension of the bicycle frame are shown in Table 2. Figure 1 show the diagram of tube.

Table 1. Material properties

Aluminium Alloy 6061		
Data	Symbol	Value
Young's Modulus	E	70 GPa
Density	p	2700 kg/m³
Poisson's ratio	v	0.34

Table 2. Dimensions of main part

Part	Length (mm)	Outer Diameter (mm)	Inner Diameter (mm)
Top tube	50	40	38
Head tube	10	40	38
Down tube	60	40	38
Seat tube	40	40	38
Seat stays	45	20	0
Chain stays	42.5	20	0

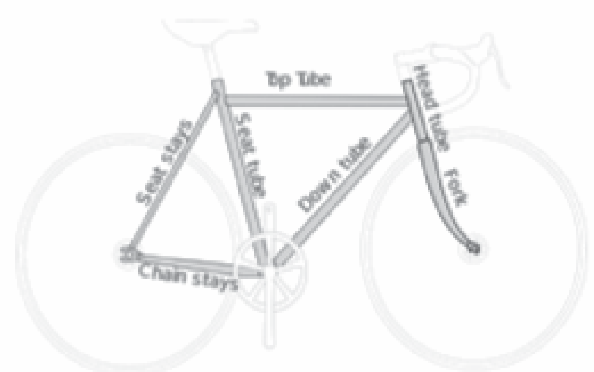

Figure 1. Diagram of tube

3. METHOD AND IMPLEMENTATION

a. Analytical method – Differential equation

In this article, we use Laplace transform to analyze the behavior of Bicycle frame systems, such as structures' vibrations, the pendulum's motion, and system dynamics.

It is used to solve partial differential equations. It transforms differential equations into algebraic equations, which are easier to solve.

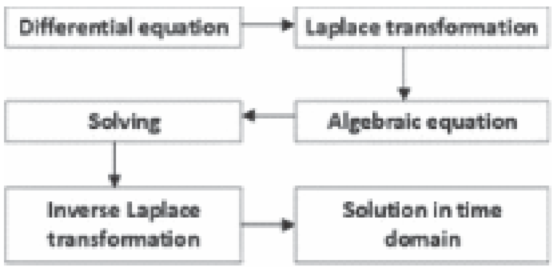

Figure 2. Laplace transform process

Mechanical dynamic system includes 3 basic elements:

- Inertia elements (mass, moment of inertia) store kinetic energy.

- Spring elements store potential energy. These elements relate with elastic force (or torque) obstructing deformation or displacement, restoring to non deformed state.

- Energy dissipating damper relates viscous friction resists motion.

The bicycle 4 DOF model consisting of front and rear absorbers shock, front and rear wheels system is illustrated as follows:

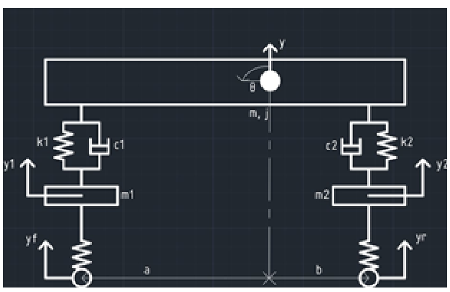

Figure 3. The 4 DOF bicycle frame

The fork with stiffness k_1, and damping coefficient c_1, on the other hand, k_2 and c_2 represent the stiffness and damping coefficient of the seat stays. The front wheel and rear wheels have the stiffness "k_f and k_r", respectively. The mass "m" is the mass of the frame and the driver, when mass "m_1, m_2" is the mass of front and rear wheels. "y_1 and y_2" are the displacements of the corresponding wheels, "y" is the displacements of seat position, "θ" is the rotation of seat place.

The free body diagram for 3 points mass:

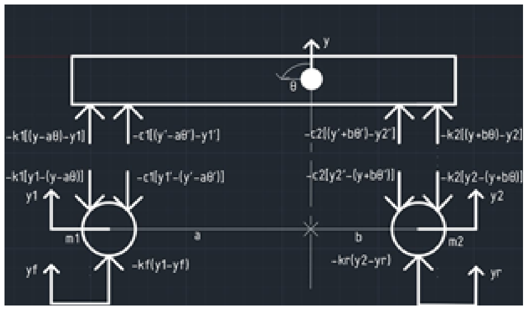

Figure 4. The free body diagram of 3 points mass

Apply the Newton II and Euler II laws, we get the equations of motion of 3 points mass:

$$
\begin{aligned}
m_1\ddot{y}_1 &= -k_1\big(y_1-(y-a\theta)\big)-c_1\big(\dot{y}_1-(\dot{y}-a\dot{\theta})\big)-k_f(y_1-y_f)\\
m_2\ddot{y}_2 &= -k_2\big(y_2-(y+b\theta)\big)-c_2\big(\dot{y}_2-(\dot{y}+b\dot{\theta})\big)-k_r(y_1-y_r)\\
m\ddot{y} &= k_1\big(y_1-(y-a\theta)\big)+c_1\big(\dot{y}_1-(\dot{y}-a\dot{\theta})\big)+k_2\big(y_2-(y+b\theta)\big)+c_2\big(\dot{y}_2-(\dot{y}+b\dot{\theta})\big)\\
j\ddot{\theta} &= -a\times[k_1\big(y_1-(y-a\theta)\big)+c_1\big(\dot{y}_1-(\dot{y}-a\dot{\theta})\big)]+b\times[k_2\big(y_2-(y+b\theta)\big)\\
&\quad +c_2\big(\dot{y}_2-(\dot{y}+b\dot{\theta})\big)]
\end{aligned}
$$

The equation of natural frequency ω_n and damping ratio ξ:

$$\xi = \frac{c}{2\sqrt{k\times m}} \tag{1}$$

$$\omega_n = \sqrt{\frac{k}{m}} \tag{2}$$

☞

According to [10], the oscillation frequency for bicycle's wheels is $f_n = 40 \div 60$ Hz, which also equal to $\omega_n = 2\pi f_n = 250 \div 380$ rad/s. While the oscillation frequency for bicycle's frame is $f_n = 80 \div 90$ Hz, which is also equal to $\omega_n = 2\pi f_n = 500 \div 565$ rad/s.

In addition, when designing suspension system, the required damping ratio ξ must be inside the range ω = (0;1). However, to prevent the vibration suddenly disappeared, which should be against in vehicle suspension, the damping ratio is designed not too close to the value of 1.

To avoid resonance, the natural frequency for bicycle's front absorber shock and rear absorber shock must be differently chosen. Therefore, the natural frequency of ω = 500 rad/s (front dropout), ω = 530 rad/s (for rear dropout) and ξ = 315 rad/s (for bicycle's wheels) are chosen. Based on equation (2), the stiffness of front and rear wheels, dropouts can be found.

In this research, the damping ratio value of ξ = 0,04 is chosen. According to equation (1), the damping coefficient of front and rear dropouts can be defined.

After getting all value of parameters, we get the table bellow.

Table 3. Proposed input parameters

Name	Parameter	Value	Unit
Mass of front & rear wheels	m_1, m_2	1	kg
Mass of driver & bicycle frame	m	103	kg
Distance from sitting to front & rear wheels	a, b	0.3	m
Damping coefficient of the front & rear dropouts	c_1 c_2	4100 4400	Ns/m
Stiffness of the front & rear dropouts	k_1 k_2	260.000 290.000	N/m
Stiffness of the front & rear wheels	k_f k_r	63.000	N/m
Inertial moment of bicycle frame	j	20	kg.m²

From the system equations of motion of 3 points mass, we take a Laplace transform:

$$\xrightarrow{\mathcal{L}}$$

$$m_1 Y_1 s^2 = -k_1\big(Y_1 - (Y - a\theta)\big) - c_1\big(Y_1 s - (Ys - a\theta s)\big) - k_f\big(Y_1 - Y_f\big)$$

$$m_2 Y_2 s^2 = -k_2\big(Y_2 - (Y + b\theta)\big) - c_2\big(\dot{y}_2 - (Ys + b\theta s)\big) - k_r\big(Y_1 - Y_r\big)$$

$$mYs^2 = k_1\big(Y_1 - (Y - a\theta)\big) + c_1\big(Y_1 s - (Ys - a\theta s)\big) + k_2\big(Y_2 - (Y + b\theta)\big) + c_2\big(\dot{y}_2 - (Ys + b\theta s)\big)$$

$$j\theta s^2 = -a \times \big[k_1\big(Y_1 - (Y - a\theta)\big) + c_1\big(Y_1 s - (Ys - a\theta s)\big)\big] + + b \times \big[k_2\big(Y_2 - (Y + b\theta)\big) + c_2\big(\dot{y}_2 - (Ys + b\theta s)\big)\big]$$

After getting the algebraic equation above, we solving it by MATLAB software. We put 4 Laplace transformed equations into MATLAB and using the code to solve them in the Sinusoidal Road Profile situation:

```
%================================
% Data
%================================
m1 = 1;        %Mass of front wheel (kg)
m2 = 1;        %Mass of rear wheel (kg)
m = 103;       %Mass of bicycle frame (kg)
```

```
a = 0.9;          %distance from sitting to front
                  wheel (m)
b = 0.3;          %distance from sitting to rear
                  wheel (m)
c1 = 4100;        %damping coefficient of the
                  front absorber (Nm/s)
c2 = 4400;        %damping coefficient of the
                  rear absorber (Nm/s)
k1 = 260000;      %stiffness of the front absorber
                  (N/m)
k2 = 290000;       %stiffness of the rear absorber
                  (N/m)
kf = 66000;       %stiffness of the front wheel
                  (N/m)
kr=66000;         %stiffness of the rear wheel
                  (N/m)
j = 20;           %inertial moment of bicycle
                  frame (kg.m^2)
v = 5;            %velocity of bicycle (m/s)
%================================
% laplace transform differential equations
%================================
syms Y Y1 Y2 TETA s t
yf= 0.05/2*sin(2*pi*5*t);
[Yf]=laplace(yf)
yr= 0.05/2*sin(2*pi*5*t-v);
[Yr]=laplace(yr)
eq1 = m1*Y1*s^2 == -k1*(Y1-(Y-a*TETA))-
c1*(Y1*s-(Y*s-a*TETA*s))-kf*(Y1-Yf);
eq2 = m2*Y2*s^2 == -k2*(Y2-(Y+b*TETA))-
c2*(Y2*s-(Y*s+b*TETA*s))-kr*(Y2-Yr);
eq3 = m*Y*s^2 == k1*(Y1-
(Y-a*TETA))+c1*(Y1*s-
(Y*s-a*TETA*s))+k2*(Y2-
(Y+b*TETA))+c2*(Y2*s-(Y*s+b*TETA*s));
eq4 = j*TETA*s^2 == -a*(k1*(Y1-
(Y-a*TETA))+c1*(Y1*s-(Y*s-
a*TETA*s)))+b*(k2*(Y2-
(Y+b*TETA))+c2*(Y2*s-(Y*s+b*TETA*s)));
%================================
% solve the system of laplace equations
%================================
[Y,Y1,Y2,TETA] = solve(eq1,eq2,eq3,eq4,Y,Y
1,Y2,TETA)
%================================
% ilaplace equations
%================================
syms s t
[y]= ilaplace (Y)
[y1]= ilaplace (Y1)
[y2]= ilaplace (Y1)
[teta]= ilaplace (TETA)
%================================
%Plot the seat displacement
%================================
fplot(y)
title('seat displacement');
xlim([0 10])
xlabel('Time');
ylabel('m');
%================================
%Plot the seat acceleration
%================================
fplot(diff(y,2))
title('seat acceleration');
xlim([0 10])
xlabel('Time');
ylabel('m/s^2');
```

After run the MATLAB code, we can get the solutions for the equations of displacement of bicycle's wheels, seat position, etc. In this article, we pay more attention about seat displacement and seat acceleration so we plot it below.

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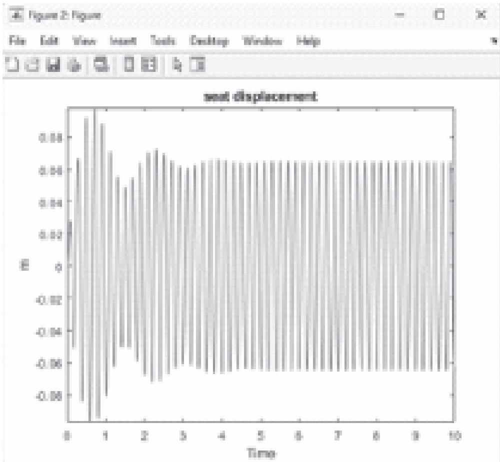

Figure 5. Seat displacement by MATLAB

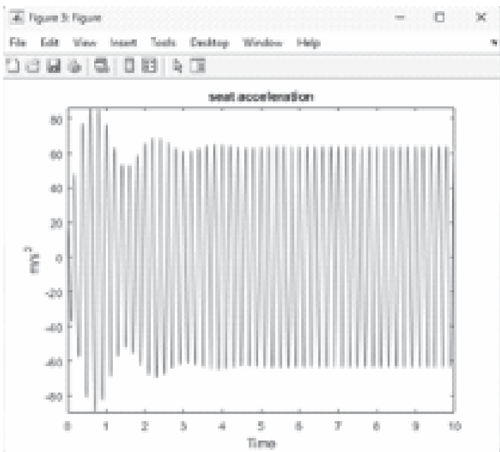

Figure 6. Seat acceleration by MATLAB

b. MATLAB Simulink method

Firstly, we consider the sinusoidal road profile, which means the vehicle is subjected under harmonic excitation. The sinusoidal road profile applied to the bicycle's wheel is determined by using equation according to [2]:

$$x_0(t) = \frac{d_2}{2}\sin\left(\frac{2\pi vt}{d_1}\right)$$

With d_1 and d_2 is the length and the height of the sine wave. In this simulation, the parameters of $d_1 = 1$m and $d_2 = 0.1$m are chosen. $v = 5$m/s denotes the velocity of the bicycle.

Figure 7. Set up for the Sinusoidal road profile

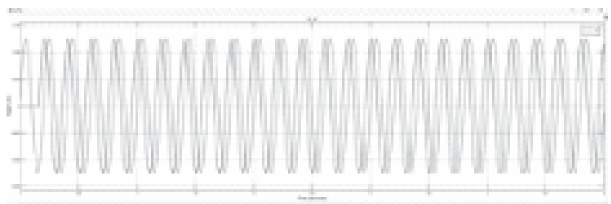

Figure 8. Sinusoidal road profile input signal

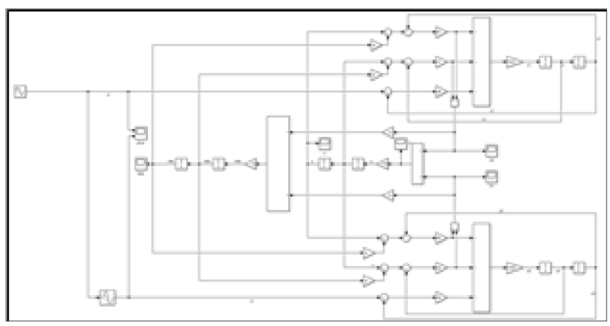

Figure 9. SIMULINK model

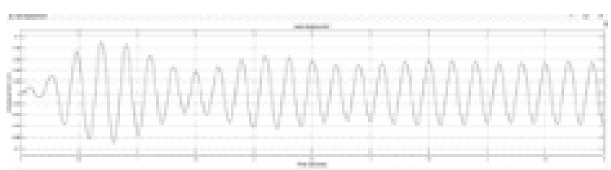

Figure 10. Seat displacement

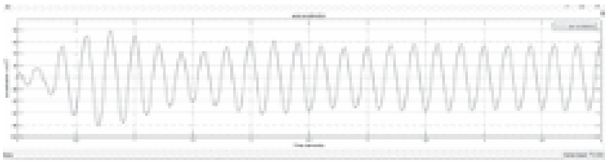

Figure 11. Acceleration of seat

4. CONCLUSION

After simulation, these above results prove that in harmonic excitation, the excitation is reduced during the time. As can be seen in figure 10, the maximum seat displacement is approximately 0.09m at t = 0.6s. Thanks to the damping ratio as well as damping coefficient, the seat displacement is reduced considerably. From 0s to 2.5s, seat displacement moves according to the natural nature of bicycle frame (natural frequency, damping ratio, etc.). After 2.5s, the seat displacement gradually becomes stable and become harmonic oscillation due to forced oscillation of the sinusoidal wave road profile, similarly for the remaining results. ❖

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