MULTI-OBJECTIVE OPTIMIZATION AND DECISION-MAKING IN ULTRASONIC VIBRATION-ASSISTED EDM OF HARDOX 500 USING NSGA-II COUPLED WITH SAW, MARCOS, AND TOPSIS

TỐI ƯU HÓA ĐA MỤC TIÊU VÀ RA QUYẾT ĐỊNH TRONG QUÁ TRÌNH GIA CÔNG XUNG ĐIỆN THÉP HARDOX 500 CÓ TRỢ GIÚP RUNG SIÊU ÂM SỬ DỤNG THUẬT TOÁN NSGA-II KẾT HỢP VỚI CÁC PHƯƠNG PHÁP SAW, MARCOS VÀ TOPSIS

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ABSTRACT

This paper presents a hybrid multi-objective optimization and decision-making framework for improving the performance of ultrasonic vibration-assisted electrical discharge machining (UV-EDM) of HARDOX 500 steel. The Non-dominated Sorting Genetic Algorithm II (NSGA-II), assisted by Gaussian Process Regression (GPR) surrogate models, was employed to simultaneously maximize the material removal rate (MRR) and minimize the surface roughness (Ra). The resulting Pareto front demonstrated a clear trade-off between machining productivity and surface quality. To identify the most balanced solution among the Pareto-optimal points, three multi-criteria decision-making (MCDM) techniques – Simple Additive Weighting (SAW), Measurement of Alternatives and Ranking according to Compromise Solution (MARCOS), and Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) – were applied. The Spearman correlation analysis showed a perfect agreement between SAW and MARCOS (ρ = 1.000) and a high correlation (ρ ≈ 0.903) between these two methods and TOPSIS. The integration of NSGA-II with comparative MCDM analysis provides a reliable and systematic approach for selecting optimal machining parameters, enabling enhanced efficiency and surface finish in UV-EDM of hardened steels.

Keywords: *Ultrasonic vibration-assisted EDM; HARDOX 500 steel; NSGA-II; Multi-criteria decision-making (MCDM); SAW-MARCOS-TOPSIS.*

TÓM TẮT

Bài báo trình bày mô hình tích hợp giữa tối ưu hóa đa mục tiêu và ra quyết định nhằm nâng cao hiệu quả gia công của quá trình gia công xung điện có trợ giúp rung siêu âm (UV-EDM) trên vật liệu thép HARDOX 500. Thuật toán NSGA-II kết hợp với mô hình thay thế GPR được sử dụng để đồng thời tối đa hóa năng suất bóc tách vật liệu (MRR) và tối thiểu hóa độ nhám bề mặt (Ra). Đồ ☞

thị Pareto thu được thể hiện rõ sự đánh đổi (trade-off) giữa năng suất và chất lượng bề mặt. Nhằm lựa chọn nghiệm cân bằng tối ưu nhất trong tập nghiệm Pareto, ba phương pháp ra quyết định đa tiêu chí (MCDM) gồm SAW, MARCOS và TOPSIS đã được áp dụng. Kết quả phân tích tương quan Spearman cho thấy mức độ tương đồng tuyệt đối giữa SAW và MARCOS (ρ = 1.000) và tương quan rất cao (ρ ≈ 0.903) giữa hai phương pháp này với TOPSIS. Sự kết hợp giữa NSGA-II và phân tích MCDM so sánh đã chứng minh khả năng hỗ trợ ra quyết định hiệu quả, giúp lựa chọn bộ thông số UV-EDM tối ưu, qua đó cải thiện đồng thời năng suất và chất lượng bề mặt khi gia công thép có độ cứng cao.

Từ khóa: *Gia công tia lửa điện có hỗ trợ rung động siêu âm; Thép HARDOX 500; NSGA-II; Ra quyết định đa tiêu chí (MCDM); SAW-MARCOS-TOPSIS.*

1. INTRODUCTION

Ultrasonic vibration-assisted electrical discharge machining (UV-EDM) has emerged as an advanced hybrid machining process capable of improving the efficiency and stability of conventional EDM by superimposing high-frequency vibrations on either the tool or the workpiece. This technique effectively enhances debris removal, dielectric circulation, and discharge uniformity, leading to better surface quality and higher machining speed. Several researchers have investigated the fundamental mechanisms of debris motion, bubble dynamics, and spark discharge under ultrasonic assistance. For instance, Wang et al. [1] and Zhang et al. [2] reported that the introduction of longitudinal or torsional ultrasonic vibration significantly improves debris ejection in deep-hole EDM, suppressing taper and recast layer formation. Similarly, Chenxue et al. [3] observed the cavitation and bubble behavior generated during discharge, confirming the enhancement of dielectric flushing and discharge stability.

Further studies have developed novel ultrasonic vibration modes and tool designs to improve EDM performance. Yin et al. [4] proposed a longitudinal–torsional ultrasonic vibration (LTV) electrode configuration for micro-EDM, achieving more uniform spark distribution and reduced tool wear. Li et al. [5, 6] introduced a two-dimensional ultrasonic circular vibration (UCV) technique to enhance surface integrity and reduce micro-hole dimensional deviation. Hirao et al. [7] demonstrated that applying ultrasonic vibration to the tool electrode can substantially increase the material removal rate (MRR) by improving discharge frequency and stability. Lei et al. [8] further explored UV-EDM for enclosed microgrooves, achieving improved flushing efficiency and consistent micro-feature geometry. These studies collectively highlight the capability of ultrasonic vibration to overcome the limitations of debris stagnation and unstable discharge in conventional EDM.

Recent investigations have also extended UV-EDM applications to hard and difficult-to-machine materials. Han et al. [9] examined ultrasonic vibration-assisted powder-mixed EDM for TiN ceramics, observing enhanced MRR and improved micro-crack resistance. Dong et al. [10] employed vertical ultrasonic vibration in EDM and demonstrated its effect on discharge energy distribution and crater morphology, while Wang et al. [11] analyzed cavitation-induced surface generation mechanisms, emphasizing the dominant role of ultrasonic amplitude in modifying the molten pool dynamics. Despite these advances, most existing studies have primarily focused on single-objective optimization – typically maximizing MRR or minimizing Ra – without addressing the multi-objective trade-off between productivity and surface quality that is critical for industrial UV-EDM of hardened steels.

To handle this trade-off effectively, evolutionary algorithms such as the Non-dominated Sorting Genetic Algorithm II (NSGA-II) have been increasingly adopted for multi-objective optimization in EDM and its hybrid variants. Xu et al. [12] successfully applied NSGA-II to optimize process parameters in ultrasonic vibration-assisted micro-EDM of Ti-6Al-4V, obtaining a well-distributed Pareto front between MRR and tool wear ratio. However, although such optimization methods can generate sets of non-dominated solutions, selecting the most appropriate solution from the Pareto front remains challenging. This decision-making stage requires systematic ranking techniques that can evaluate and prioritize the alternatives according to multiple conflicting criteria.

Multi-criteria decision-making (MCDM) methods, including Simple Additive Weighting (SAW), Measurement of Alternatives and Ranking according to Compromise Solution (MARCOS), and Technique for Order Preference by Similarity to Ideal Solution (TOPSIS), have recently gained attention for post-Pareto evaluation and decision support in manufacturing optimization. However, their comparative consistency and reliability when applied to UV-EDM have not been systematically analyzed. This research gap motivates the present study, which aims to develop a hybrid NSGA-II–MCDM framework for optimizing and selecting the best machining parameters in ultrasonic vibration-assisted EDM of HARDOX 500 steel, targeting the simultaneous maximization of MRR and minimization of Ra.

In this study, Gaussian Process Regression (GPR) surrogate models were first constructed to predict MRR and Ra, followed by NSGA-II optimization to generate the Pareto-optimal front. The obtained solutions were subsequently ranked using SAW, MARCOS, and TOPSIS to determine the most balanced machining condition. Finally, a Spearman correlation analysis was performed to assess the consistency among the MCDM methods, ensuring the robustness of the decision-making results. The proposed approach provides a systematic and data-driven framework for multi-objective optimization and decision support in UV-EDM of hardened steels.

2. METHODOLOGY

2.1. Multi-objective optimization using NSGA-II

The NSGA-II is a population-based evolutionary algorithm widely employed for solving multi-objective optimization problems due to its ability to achieve both convergence and diversity within the Pareto front. In this study, NSGA-II was utilized to determine the optimal machining parameters of ultrasonic vibration-assisted EDM (UV-EDM) for HARDOX 500 steel with two conflicting objectives: maximizing the material removal rate (MRR) and minimizing the surface roughness (Ra).

The optimization process was implemented through the following main steps: ☞

1) Initialization: An initial population of candidate solutions was generated within the experimental design range of the input machining parameters (such as discharge current, pulse-on time, pulse-off time, and ultrasonic amplitude).

2) Fitness evaluation: The performance values of each candidate were predicted using Gaussian Process Regression (GPR)-based surrogate models developed for MRR and Ra.

3) Non-dominated sorting: All individuals were classified into multiple Pareto fronts according to the dominance relationship among solutions.

4) Diversity preservation: A crowding distance measure was calculated to maintain a well-distributed population along the front.

5) Genetic operations: Crossover and mutation operators were applied to generate new offspring from the parent population.

6) Selection: The combined parent and offspring populations were ranked, and the best individuals were selected based on Pareto rank and crowding distance.

7) Termination: The algorithm iteratively repeated these steps until the convergence criterion or the predefined number of generations was reached.

The final output of NSGA-II is a set of Pareto-optimal solutions representing the trade-off between the two objectives, providing the basis for subsequent decision-making analysis using MCDM techniques.

2.2. Multi-Criteria Decision-Making Methods: SAW, MARCOS, and TOPSIS

After obtaining the Pareto-optimal solutions from NSGA-II, a decision-making stage was performed to identify the most balanced machining condition among the non-dominated alternatives. For this purpose, three well-established and widely used multi-criteria decision-making (MCDM) techniques – SAW, MARCOS, and TOPSIS – were employed.

These MCDM methods share the same objective: to evaluate and rank the Pareto-optimal solutions based on multiple performance criteria (e.g., MRR as a benefit criterion and Ra as a cost criterion). The general procedure can be summarized as follows:

1) Normalization: Convert the performance values of each alternative into dimensionless form to allow comparison across criteria.

2) Weight assignment: Assign importance weights to each criterion according to their relative significance.

3) Computation of ranking indices: Aggregate the normalized and weighted criteria to compute a ranking score for each alternative.

4) Final ranking: Sort the alternatives from best to worst according to the computed scores.

The SAW method [13] is a simple additive approach based on the weighted sum of normalized criteria values. The MARCOS method [14] evaluates each alternative relative to both the ideal and anti-ideal solutions, allowing balanced ranking outcomes. The

TOPSIS method [15] determines the proximity of each alternative to the ideal and anti-ideal points in the decision space.

All three methods are well-recognized for their simplicity, stability, and interpretability. In the present study, they were applied independently to the Pareto solutions obtained from NSGA-II to support the final decision-making process. The detailed mathematical formulations of these MCDM methods are not presented here, as they are thoroughly described in the referenced literature [13-15].

3. EXPERIMENTAL WORK

All experiments were conducted on a Sodick A30 CNC EDM system modified to incorporate a high-power ultrasonic vibration module, as shown in Figure 1. The ultrasonic integration aimed to enhance spark stability, improve dielectric flushing, and facilitate effective debris removal during the machining of HARDOX 500 steel.

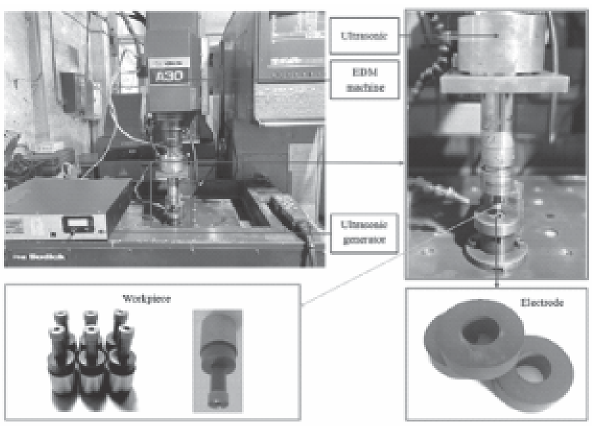

Figure 1. Experimental setup

Ultrasonic vibrations were supplied by an MPI WG-3000 generator (MPI Ultrasonics, Switzerland; 3000 W) and transmitted through an RPS-5020-4Z transducer operating at 20 kHz with a nominal power of 2000 W. A custom titanium horn was designed to efficiently transfer vibrational energy to the machining zone, ensuring stable oscillation at the tool–workpiece interface and uniform spark distribution.

The tool electrode was made of HK0 graphite, selected for its high electrical conductivity and wear resistance. The dielectric medium was Diel MS 7000 (Total, France), chosen for its stable discharge characteristics and high thermal endurance. The workpiece material was HARDOX 500, a quenched and tempered steel with superior hardness and abrasion resistance. Cylindrical specimens were used to maintain uniform flushing and surface measurement conditions.

Two output responses were evaluated: material removal rate (MRR) and surface roughness (Ra). MRR was determined by the gravimetric method using an analytical balance (resolution = 0.1 mg) according to:

$$MRR = \frac{m_{before} - m_{after}}{t} \cdot 3600 \qquad (1)$$

Where m_{before} and m_{after} are the workpiece masses before and after machining (g), and t is the machining time (s). Each specimen was cleaned and dried before weighing to ensure measurement consistency.

Surface roughness (Ra) was measured using a Mitutoyo SV-3100 tester. Three readings were taken along the cylindrical surface of each specimen, and their mean value was reported as the representative Ra.

The experimental design followed the Box–Behnken Design (BBD), suitable for multi-factor response surface modeling with a reduced number of runs. Five input parameters ☞

were investigated – ultrasonic amplitude (A), pulse-on time (T_{on}), pulse-off time (T_{off}), discharge current (IP), and servo voltage (SV) – each at three levels (low, medium, high) determined through preliminary trials within the operating range of the machine.

The complete experimental matrix and measured responses (MRR and Ra) are summarized in Table 1.

Table 1. Experimental plan and output results (MRR and Ra)

No.	A (µm)	T_{on} (µs)	T_{off} (µs)	IP (A)	SV (V)	MRR (g/h)	Ra (µm)
1	1.2	8	12	10	5	3.346	3.353
2	1.2	16	12	10	5	1.482	6.569
3	5.2	8	12	10	5	3.119	3.198
4	5.2	16	12	10	5	1.228	5.711
5	3.2	12	8	5	5	1.210	7.220
…							
21	3.2	8	8	10	5	4.216	3.430
22	3.2	8	16	10	5	3.290	3.256
23	3.2	16	8	10	5	1.391	7.280
24	3.2	16	16	10	5	1.360	5.313
25	1.2	12	12	5	5	1.351	6.730
…							
44	3.2	12	12	10	5	3.799	5.032
45	3.2	12	12	10	5	3.764	5.455
46	3.2	12	12	10	5	3.775	4.785

4. RESULTS AND DISCUSSION

4.1. Multi-objective optimization results

The NSGA-II algorithm, assisted by GPR surrogate models, successfully generated a well-distributed Pareto front representing the trade-off between material removal rate (MRR) and surface roughness (Ra) in ultrasonic vibration-assisted EDM (UV-EDM) of HARDOX 500 steel. As shown in Figure 2, the Pareto front demonstrates an inverse relationship between the two objectives – an increase in MRR generally corresponds to a rise in Ra. This trend reflects the intrinsic machining trade-off in EDM: higher discharge energy enhances material melting and ejection, thereby improving productivity but simultaneously deteriorating surface finish.

The dispersion of Pareto-optimal points indicates that NSGA-II effectively maintained population diversity while converging toward the global Pareto front. The GPR-assisted approach proved computationally efficient and accurate, allowing reliable prediction of both objectives within the experimental domain. These results confirm the suitability of the NSGA-II–GPR framework for identifying balanced machining conditions in hybrid EDM processes.

The obtained Pareto solutions form the decision space for the next analytical stage, where multi-criteria decision-making (MCDM) methods are applied to determine the most favorable machining setting that simultaneously ensures high MRR and low Ra.

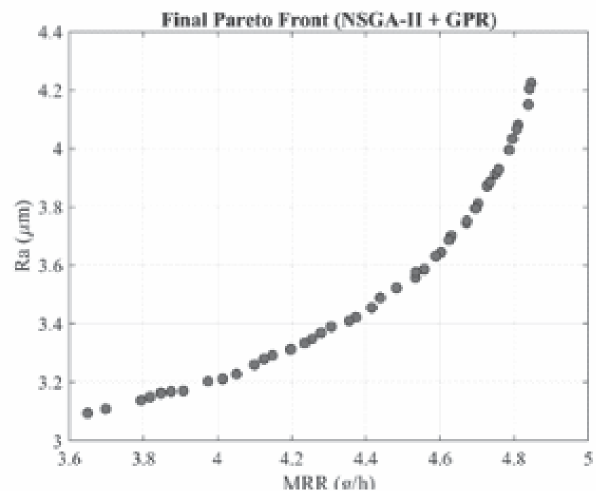

Figure 2. Pareto-optimal front obtained by NSGA-II for ultrasonic vibration-assisted EDM of HARDOX 500 steel

4.2. Comparative ranking using MCDM methods

Three established MCDM methods – SAW, MARCOS, and TOPSIS – were independently applied to rank the Pareto-optimal solutions based on the dual criteria of MRR (benefit) and Ra (cost). All three approaches successfully identified the top-performing alternatives, with minor variations in ranking order due to differences in their normalization and aggregation strategies.

To evaluate the consistency among the three methods, the Spearman rank correlation coefficients were computed and are presented in Figure 3. The results reveal perfect agreement between SAW and MARCOS (ρ = 1.000), indicating identical ranking patterns, while both exhibited strong correlation with TOPSIS (ρ ≈ 0.903). This high degree of consistency confirms the robustness of the ranking process and demonstrates that all three MCDM techniques lead to comparable decision outcomes.

Among the methods, MARCOS showed superior interpretability in balancing the trade-off between MRR and Ra, while SAW offered the most computational simplicity. TOPSIS produced slightly different rankings due to its geometric distance-based formulation but remained in strong agreement with the other two approaches. The overall analysis verifies that the integration of NSGA-II with comparative MCDM methods provides a reliable framework for post-Pareto decision support in UV-EDM optimization.

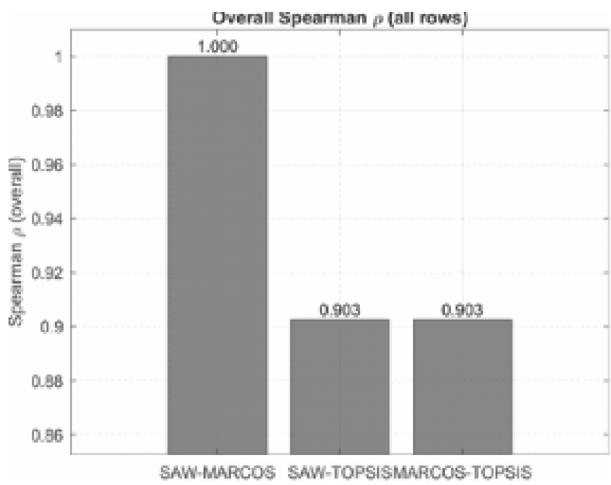

Figure 3. Spearman rank correlation coefficients among the three MCDM methods (SAW, MARCOS, and TOPSIS) applied to the Pareto-optimal solutions

Furthermore, all three MCDM methods – SAW, MARCOS, and TOPSIS – identified the same optimal solution from the Pareto front, confirming the strong consistency observed in the Spearman analysis. The selected optimal ☞

parameter combination was A = 4.17µm, T_{on} = 9.55µs, T_{off} = 8.00µs, IP = 11.71A, and SV = 5.36V, which yielded Ra = 4.22µm and MRR = 4.85g/h.

This convergence of results across all methods further validates the robustness of the proposed NSGA-II–MCDM framework and enhances confidence in the final decision-making outcome.

4.3. Discussion of optimal decision and process implications

The combined NSGA-II–MCDM framework enables a two-stage optimization strategy: NSGA-II efficiently generates the Pareto front, and MCDM methods refine the final selection according to specific performance priorities. The results highlight that ultrasonic vibration significantly improves EDM efficiency through enhanced debris evacuation and spark stability, allowing higher MRR values to be achieved without severely compromising surface integrity.

For practical implementation, the selected optimal condition – identified consistently by all three MCDM approaches – represents the most balanced trade-off between productivity and surface quality. The methodology established in this work provides a systematic, data-driven, and adaptable framework that can be extended to other hybrid or powder-mixed EDM processes involving multiple conflicting performance criteria.

5. CONCLUSION

This study proposed an integrated framework combining the NSGA-II and MCDM methods to optimize and select machining parameters in UV-EDM of HARDOX 500 steel. Two conflicting objectives – maximizing MRR and minimizing Ra – were simultaneously optimized using NSGA-II assisted by GPR surrogate models.

The algorithm effectively generated a well-distributed Pareto front representing the trade-off between productivity and surface quality. Subsequently, three widely used MCDM techniques – SAW, MARCOS, and TOPSIS – were applied to rank the Pareto-optimal solutions. Spearman correlation analysis indicated a perfect agreement between SAW and MARCOS (ρ = 1.000) and a high correlation with TOPSIS (ρ ≈ 0.903), confirming the consistency and robustness of the decision-making process.

All three MCDM methods converged to the same optimal solution (A = 4.17µm, T_{on} = 9.55µs, T_{off} = 8.00µs, IP = 11.71A, SV = 5.36V), corresponding to Ra = 4.22µm and MRR = 4.85 g/h. This consistency validates the reliability of the proposed NSGA-II–MCDM integration in identifying balanced machining conditions.

Acknowledgments:

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