

MULTI-OBJECTIVE OPTIMIZATION OF A TWO-STAGE HELICAL GEAR REDUCER WITH A SPLIT OUTPUT STAGE USING NSGA-II AND EDAS: MINIMIZING CROSS-SECTIONAL AREA AND MAXIMIZING EFFICIENCY

TỐI ƯU HÓA ĐA MỤC TIÊU HỘP GIẢM TỐC BÁNH RĂNG CẤP CHẬM
PHÂN ĐÔI BẰNG THUẬT TOÁN NSGA-II VÀ EDAS: GIẢM THIỂU DIỆN TÍCH
MẶT CẮT NGANG VÀ TỐI ĐA HÓA HIỆU SUẤT

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ABSTRACT

This study presents a multi-objective optimization approach for a two-stage helical gear reducer with a split output stage to simultaneously minimize the gearbox cross-sectional area and maximize transmission efficiency. A mathematical design model incorporating geometric constraints, transmission relationships, strength conditions, and efficiency characteristics was developed. The NSGA-II algorithm was employed to generate Pareto-optimal solutions, while the EDAS method was used to identify the most balanced design alternatives.

The results revealed a clear trade-off between compactness and efficiency. As the total transmission ratio increased, the Pareto fronts shifted toward larger cross-sectional areas and lower efficiencies. A strong linear relationship was also observed between the total transmission ratio and the optimal first-stage transmission ratio, with coefficients of determination exceeding 0.99. This finding provides valuable insight for preliminary transmission ratio allocation in split-stage gear reducers.

The proposed NSGA-II–EDAS framework proved effective in exploring the feasible design space and selecting practical compromise solutions for compact and high-efficiency gearbox systems.

Keywords: Helical gear reducer; Split output stage; Multi-objective optimization; NSGA-II; EDAS; Pareto optimization.

TÓM TẮT

Nghiên cứu này trình bày phương pháp tối ưu hóa đa mục tiêu cho hộp giảm tốc bánh răng trụ răng nghiêng hai cấp có cấp ra phân đôi nhằm đồng thời giảm nhỏ nhất diện tích mặt cắt ngang

và tăng lớn nhất hiệu suất truyền động của hộp giảm tốc. Một mô hình thiết kế toán học có xét đến các ràng buộc hình học, quan hệ tỷ số truyền, điều kiện bền và đặc tính hiệu suất đã được xây dựng. Thuật toán NSGA-II được sử dụng để tạo tập nghiệm Pareto tối ưu, trong khi phương pháp EDAS được áp dụng để lựa chọn các phương án thiết kế cân bằng nhất.

Kết quả cho thấy tồn tại sự đánh đổi rõ rệt giữa tính gọn nhẹ và hiệu suất truyền động. Khi tỷ số truyền tổng tăng, các đường Pareto dịch chuyển theo hướng diện tích mặt cắt ngang lớn hơn và hiệu suất thấp hơn. Ngoài ra, một quan hệ tuyến tính rất chặt chẽ giữa tỷ số truyền tổng và tỷ số truyền tối ưu của cấp nhanh đã được xác định với hệ số xác định lớn hơn 0,99. Kết quả này cung cấp cơ sở hữu ích cho việc phân phối tỷ số truyền sơ bộ trong hộp giảm tốc phân đôi.

Khung tối ưu hóa NSGA-II–EDAS đề xuất cho thấy hiệu quả cao trong việc khám phá không gian nghiệm khả thi và lựa chọn các nghiệm thỏa hiệp phù hợp cho thiết kế hộp giảm tốc gọn nhẹ và hiệu suất cao.

Từ khóa: Hộp giảm tốc bánh răng trụ răng nghiêng; Cấp chậm phân đôi; Tối ưu hóa đa mục tiêu; NSGA-II; EDAS; Tối ưu Pareto.

1. INTRODUCTION

Gear transmission systems are widely used in industrial machinery because of their high efficiency, compact structure, and reliable operation. Among them, two-stage helical gear reducers are especially attractive due to their high load-carrying capacity and smooth transmission characteristics [1-4]. In modern mechanical systems, increasing demands for compactness and energy efficiency have promoted extensive research on gearbox optimization methods [5-7].

Traditional gearbox design is mainly based on empirical calculations and deterministic approaches, which are often inadequate for handling multiple conflicting objectives simultaneously. Therefore, multi-objective optimization techniques have become increasingly important in gearbox design problems involving trade-offs among size, efficiency, vibration, and strength requirements [6], [7]. Evolutionary algorithms, particularly the Non-dominated Sorting Genetic Algorithm

II (NSGA-II), have shown strong effectiveness in solving nonlinear multi-objective engineering problems [1-3]. NSGA-II has been successfully applied to the optimization of spur gears, planetary gear trains, and helical gearboxes [8], [9], [10], [11].

Several recent studies have focused on multi-objective optimization of two-stage helical gearboxes using different decision-making methods. Le and Vu [12] applied the Taguchi method and Grey Relational Analysis for gearbox optimization, while Dinh et al. [13], [14-17] employed TOPSIS, MARCOS, and MAIRCA methods to select compromise solutions from Pareto sets. Other studies investigated optimization objectives related to gearbox volume, bottom area, and efficiency improvement [18-22]. These studies confirmed the effectiveness of combining evolutionary optimization with multi-criteria decision-making approaches.

Despite these advances, studies on two-stage helical gear reducers with a split

output stage remain limited, particularly for the simultaneous optimization of cross-sectional area and transmission efficiency. In addition, the relationship between the total transmission ratio and optimal transmission-ratio allocation has not been thoroughly investigated.

To address these gaps, this study proposes a multi-objective optimization framework for a two-stage helical gear reducer with a split output stage using NSGA-II and the Evaluation based on Distance from Average Solution (EDAS) method. The objectives are to minimize the gearbox cross-sectional area (A_c) and maximize transmission efficiency simultaneously. NSGA-II is employed to generate Pareto-optimal solutions, while EDAS is used to identify the most balanced compromise designs. The obtained results provide useful insights for the design of compact and high-efficiency gearbox systems.

2. OPTIMIZATION PROBLEM FORMULATION

2.1. Calculation of gearbox cross-sectional area

For a two stage helical gearbox with a split output stage, A_c is calculated by (Figure 1):

$$A_c = (L \cdot H) \quad (1)$$

Where L and H are determined by:

$$L = d_{w11} + d_{w21}/2 + d_{w12}/2 + d_{w22} + 2 \cdot \delta \quad (2)$$

In which, $\delta = 7 \div 10$ (mm) [18]; d_{w1i} , d_{w2i} ($i=1 \div 2$) are the pitch diameter of the pinion and the gear of stage i which can be computed by [23]:

$$d_{w1i} = 2 \cdot a_{wi} / (u_i + 1) \quad (4)$$

$$d_{w2i} = 2 \cdot a_{wi} \cdot u_i / (u_i + 1) \quad (5)$$

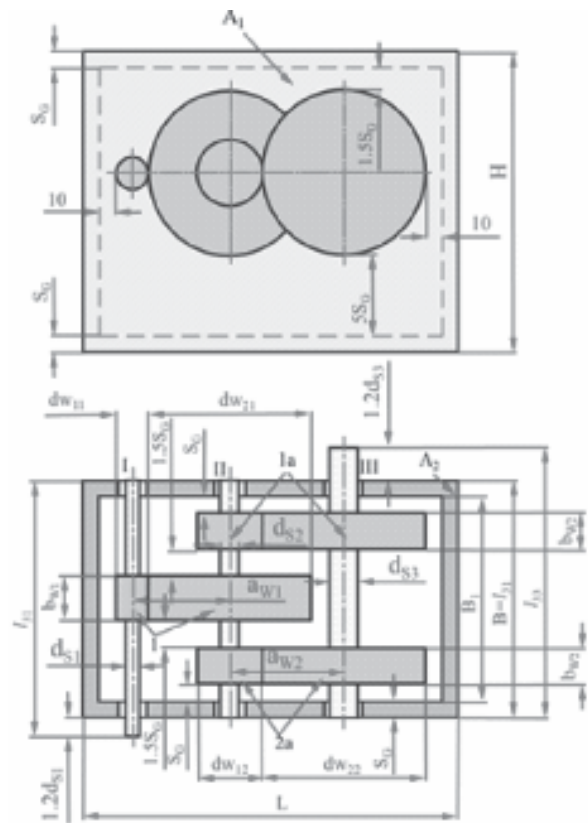


Figure 1. Schema for determination of gearbox cross-sectional area

In which, a_{wi} ($i=1 \div 2$) denotes the center distance of stage and it is found by [23]:

$$a_{wi} = k_a \cdot (u_i + 1) \cdot \sqrt[3]{T_{1i} \cdot k_{H\beta} / ([AS_i]^2 \cdot u_i \cdot X_{bai})} \quad (6)$$

Where X_{bai} represents the wheel face width coefficient of stage i th; T_{1i} ($i=1 \div 2$) is the pinion torque of stage i and it can be determined by:

$$T_{11} = \frac{T_r}{u_{gb} \cdot \eta_{hg}^2 \cdot \eta_{be}^3} \quad (7)$$

$$T_{12} = \frac{T_r}{2 \cdot u_2 \cdot \eta_{hg} \cdot \eta_{be}^2} \quad (8)$$

2.2. Calculations of gearbox efficiency

The gearbox efficiency (%) can be calculated by:

$$\eta_{gb} = 100 - \frac{100 \cdot P_l}{P_{in}} \quad (9)$$

In which, P_l denotes the total gearbox power loss which is determined by [24]:

$$P_l = P_{lg} + P_{lb} + P_{ls} + P_{zo} \quad (10)$$

The terms P_{lg} , P_{lb} , P_{ls} , and P_{zo} denote the power losses in the gears, bearings, seals, and during idle motion, respectively. These components can be determined as presented in [25].

2.3. Objectives functions

This study formulates the design problem as a bi-objective optimization task for a two-stage helical gearbox with a split output stage. The optimization aims to simultaneously minimize the gearbox cross-sectional area and maximize the gearbox efficiency. The objective functions are defined as follows:

$$\min f_1(X) = A_c \quad (11)$$

$$\min f_2(X) = \eta_{gb} \quad (12)$$

Where A_c denotes the gearbox cross-sectional area and η_{gb} represents the overall gearbox efficiency.

The design variable vector X consists of the primary geometrical parameters affecting the gearbox performance. Previous studies commonly considered five design variables, including u_1 , Xba_1 , Xba_2 , AS_1 , and AS_2 .

However, it was reported in [25] that the optimal values of AS_1 and AS_2 usually converge to their upper bounds during the optimization process. Therefore, only the three most influential and adjustable variables are selected as decision variables in the present study, namely the first-stage transmission ratio and the face width coefficients of both stages. Accordingly, the design variable vector is expressed as:

$$X = \{u_1, Xba_1, Xba_2\} \quad (13)$$

3. OPTIMIZATION METHODOLOGY

The optimization process in this study was implemented in MATLAB using the Non-dominated Sorting Genetic Algorithm II (NSGA-II). The proposed framework was developed to determine Pareto-optimal solutions for the two-stage helical gearbox with a split output stage under different total transmission ratios. The optimization simultaneously considered the minimization of gearbox cross-sectional area and the maximization of gearbox efficiency.

The optimization procedure began with the random initialization of the population within predefined lower and upper bounds of the design variables. The decision variable vector was defined as:

$$X = \{u_1, Xba_1, Xba_2\} \quad (14)$$

Where u_1 is the first-stage transmission ratio, while Xba_1 and Xba_2 are the face width coefficients of the first and second stages, respectively. The lower and upper bounds used in the optimization process were:

$$1 \leq u_1 \leq 9 \quad (15)$$

$$0.25 \leq Xba_1, Xba_2 \leq 0.40 \quad (16)$$

For each candidate solution, the objective function evaluated the gearbox cross-sectional area and transmission efficiency based on the developed mathematical model. The second-stage transmission ratio was determined from:

$$u_2 = \frac{u_h}{u_1} \quad (17)$$

Where u_h denotes the total transmission ratio of the gearbox.

The NSGA-II procedure employed non-dominated sorting and crowding-distance calculation to classify solutions and maintain population diversity. Tournament selection was used to select parent individuals for reproduction. Simulated Binary Crossover (SBX) and polynomial mutation operators were adopted to generate offspring solutions and improve the exploration capability of the search process. The crossover probability and mutation probability were set to 0.9 and 0.1, respectively, while both crossover and mutation distribution indices were chosen as 20.

In the MATLAB implementation, the population size and the maximum number of generations were set to 100 and 200, respectively. The crossover and mutation probabilities were selected as 0.9 and 0.1 to ensure an effective balance between exploration and exploitation during the optimization process. In addition, the distribution indices for the Simulated Binary Crossover (SBX) and polynomial mutation operators were both assigned a value of 20 to maintain solution diversity and improve convergence stability.

The optimization process was repeated for different total transmission ratios ranging

from 5 to 30 with an increment of 5. After each optimization run, the first Pareto front was extracted and stored. The obtained Pareto-optimal solutions were then analyzed to investigate the trade-off relationship between gearbox compactness and transmission efficiency.

To support engineering decision-making, additional filtering procedures were implemented to determine representative optimal solutions corresponding to:

- The minimum gearbox cross-sectional area;
- The highest gearbox efficiency;
- And the best efficiency-to-cross-sectional-area ratio.

To support engineering decision-making, the Evaluation based on Distance from Average Solution (EDAS) method was employed to rank the Pareto-optimal solutions and identify the most balanced compromise design. In the EDAS approach, the performance of each alternative is evaluated according to its positive and negative distances from the average solution for all considered criteria. The method provides an effective mechanism for selecting practical solutions from the Pareto set while simultaneously considering both optimization objectives. The implementation procedure of the EDAS method in this study follows the approach presented in [26].

The methodology description is based on the MATLAB implementation provided in the uploaded source code.

4. RESULTS AND DISCUSSION

4.1. Pareto Front Characteristics

The NSGA-II algorithm successfully

generated well-distributed Pareto-optimal solutions for all investigated total transmission ratios. The obtained Pareto fronts are illustrated in Figure 1. A clear trade-off relationship between gearbox cross-sectional area and transmission efficiency can be observed. In general, higher transmission efficiency is associated with larger gearbox dimensions, whereas more compact gearbox designs tend to exhibit lower efficiency values.

As the total transmission ratio increases from $u_h = 5$ to $u_h = 30$, the Pareto fronts gradually shift toward larger cross-sectional areas and lower efficiency regions. This trend is mainly caused by increased gear dimensions, contact forces, and transmission losses at higher transmission ratios. The smooth and continuous distribution of the Pareto fronts also confirms the stable convergence behavior and effective search capability of the implemented NSGA-II algorithm.

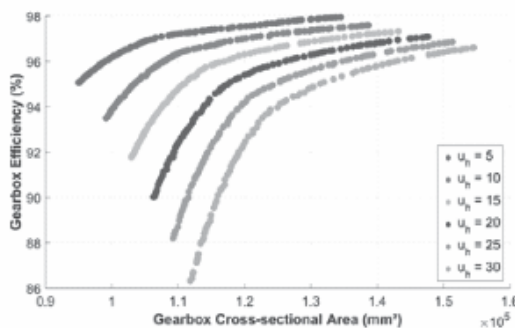


Figure 1. Pareto fronts obtained for different total transmission ratios of the two-stage helical gearbox with split output stage.

4.2. Influence of Total Transmission Ratio

The results indicate that the total transmission ratio significantly affects both gearbox compactness and efficiency. At relatively small transmission ratios, such as $u_h = 5$, the gearbox achieves high efficiency

with comparatively small cross-sectional dimensions. However, increasing the transmission ratio results in larger gearbox size and reduced efficiency due to higher meshing losses and larger gear dimensions.

These findings demonstrate that excessively large transmission ratios may considerably reduce overall gearbox performance. Therefore, suitable transmission-ratio allocation between stages is important for achieving balanced design characteristics.

4.3. Transmission Ratio Allocation Analysis

An important finding of this study is the strong linear relationship between the total transmission ratio u_h and the optimal first-stage transmission ratio u_1 . The relationships obtained from the Pareto-optimal solutions and the EDAS-selected compromise solutions are presented in Figure 2 and Figure 3, respectively. Both regression models achieved coefficients of determination greater than 0.99, indicating excellent fitting accuracy.

Figure 2, which was established using the mean values of the Pareto-optimal solutions, shows that the optimal first-stage transmission ratio increases almost linearly with increasing total transmission ratio. A similar trend can also be observed in Figure 3 for the solutions selected by the EDAS method. The corresponding regression equation obtained from the EDAS-based solutions is:

$$u_1 = 1.3604 + 0.2024u_h \quad (18)$$

With $R^2 = 0.9908$.

The results indicate that assigning a larger portion of the transmission ratio to the first stage becomes increasingly advantageous as the total transmission ratio increases. This

allocation strategy helps reduce excessive dimensional growth and transmission losses in the split output stage, thereby improving the compromise between gearbox compactness and efficiency.

Furthermore, the close agreement between the Pareto-based and EDAS-based relationships demonstrates the stability and reliability of the proposed optimization framework. The obtained empirical relationships can therefore provide useful guidance for preliminary transmission-ratio allocation in practical gearbox design applications.

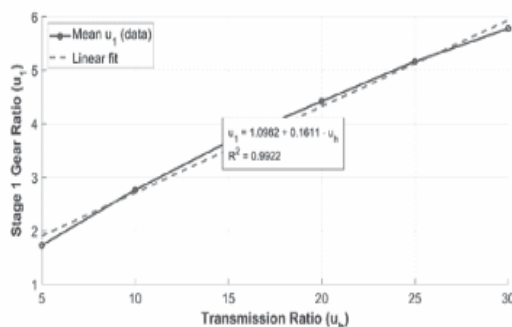


Figure 2. Linear relationship between the total transmission ratio and the mean optimal first-stage transmission ratio obtained from Pareto-optimal solutions.

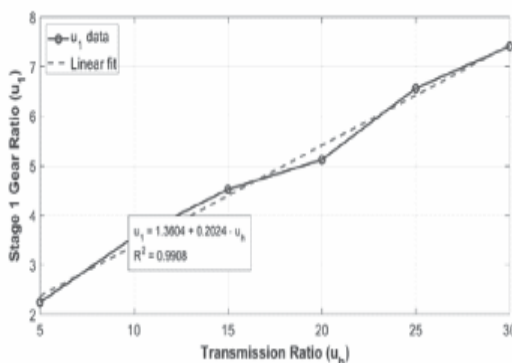


Figure 3. Linear relationship between the total transmission ratio and the first-stage transmission ratio obtained from EDAS-selected compromise solutions.

4.4. EDAS-Based Selection of Optimal Solutions

Although NSGA-II generates multiple Pareto-optimal solutions, practical gearbox design generally requires a single compromise solution. Therefore, the EDAS method was employed to rank the Pareto solutions and identify the most balanced design alternative.

The EDAS-based compromise solutions provided favorable balance between gearbox compactness and transmission efficiency. Compared with solutions optimized for only one objective, the selected designs demonstrated better overall engineering applicability and more practical performance characteristics.

4.5. Engineering Implications

The proposed NSGA-II-EDAS framework proved effective for solving the multi-objective optimization problem of the two-stage helical gearbox with split output stage. The obtained results provide useful insight into the influence of transmission-ratio allocation on gearbox performance and offer practical guidance for designing compact and high-efficiency gearbox systems.

5. CONCLUSIONS

This study proposed a multi-objective optimization framework for a two-stage helical gear reducer with a split output stage using NSGA-II and EDAS. The optimization simultaneously minimized gearbox cross-sectional area and maximized transmission efficiency. The obtained Pareto fronts revealed a clear trade-off between gearbox compactness and efficiency. Increasing the total transmission ratio resulted in larger gearbox dimensions and lower efficiency due to increased transmission

losses and gear size. A strong linear relationship between the total transmission ratio and the optimal first-stage transmission ratio was identified, with regression models achieving $R^2 > 0.99$. The results indicate that allocating a larger portion of the transmission ratio to the first stage improves the compromise between compactness and efficiency. The EDAS method effectively selected balanced compromise solutions from the Pareto set, demonstrating the practicality of the proposed optimization framework for compact and high-efficiency gearbox design. ♦

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