# SURFACE ROUGHNESS OPTIMIZATION IN ULTRASONIC VIBRATION-ASSISTED EDM USING GAUSSIAN PROCESS REGRESSION

## TỐI ƯU HÓA ĐỘ NHÁM BỀ MẶT TRONG GIA CÔNG XUNG ĐIỆN CÓ HỖ TRỢ DAO ĐỘNG SIÊU ÂM BẰNG MÔ HÌNH GAUSSIAN PROCESS REGRESSION

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**ABSTRACT**

*In this study, a Gaussian Process Regression (GPR) model was developed to accurately predict and minimize surface roughness (Ra) in ultrasonic vibration-assisted electrical discharge machining (UV-EDM). A five-factor Box-Behnken design (BBD) was employed to investigate the effects of ultrasonic amplitude (A), pulse on-time ($T_{on}$), pulse off-time ($T_{off}$), peak current (IP), and spark voltage (SV) on Ra. A second-order regression model (RSM) was initially constructed and compared with the GPR model in terms of prediction accuracy. The GPR model achieved a significantly higher coefficient of determination ($R^2 = 0.9284$) compared to RSM, demonstrating its superior capability in handling nonlinearities and noise in surface data. The optimal machining parameters were identified using fmincon in MATLAB, based on the trained GPR model. The findings confirm that GPR is an effective surrogate model for predicting and optimizing surface quality in UV-EDM, offering valuable guidance for selecting optimal process parameters.*

**Keywords:** *Surface roughness; Ultrasonic vibration-assisted EDM; Gaussian Process Regression; RSM; Optimization; Predictive modeling; Machine learning; Box-Behnken design.*

**TÓM TẮT**

*Trong nghiên cứu này, một mô hình hồi quy quá trình Gaussian (Gaussian Process Regression – GPR) đã được xây dựng nhằm dự đoán và tối ưu hóa chính xác độ nhám bề mặt (Ra) trong quá trình gia công xung điện có hỗ trợ rung động siêu âm (UV-EDM). Thiết kế thí nghiệm Box-Behnken (BBD) với năm thông số đầu vào gồm biên độ dao động siêu âm (A), thời gian phát xung ($T_{on}$), thời gian ngắt xung ($T_{off}$), dòng điện phóng điện (IP) và điện áp phóng điện (SV) đã được sử dụng để khảo sát ảnh hưởng của các thông số này đến Ra. Một mô hình hồi quy bậc hai (RSM) được xây dựng ban đầu để làm mô hình tham chiếu và được so sánh với mô hình GPR về độ chính xác dự đoán. Mô hình GPR đạt hệ số xác định cao ($R^2 = 0,9284$), vượt trội so với RSM, cho thấy khả năng mạnh mẽ của GPR trong việc mô hình hóa các đặc tính phi tuyến và nhiễu trong dữ liệu Ra. Tối ưu hóa được thực hiện bằng công cụ fmincon trong MATLAB® dựa trên mô hình GPR đã huấn luyện. Kết quả nghiên cứu khẳng định rằng GPR là một mô hình thay thế hiệu quả cho việc dự đoán và tối ưu hóa chất lượng bề mặt trong UV-EDM, đồng thời cung cấp định hướng thực tiễn cho việc lựa chọn thông số công nghệ tối ưu.*

**Từ khóa:** *Nhám bề mặt; EDM hỗ trợ rung siêu âm; Hồi quy quy trình Gaussian; RSM; Tối ưu hóa; Mô hình dự đoán; Học máy; Thiết kế Box-Behnken.*

## 1. INTRODUCTION

Ultrasonic vibration-assisted electrical discharge machining (UV-EDM) has attracted significant attention due to its ability to enhance machining performance, particularly in terms of debris removal, surface quality, and material removal rate (MRR) when compared to conventional EDM. In UV-EDM, ultrasonic vibration is superimposed onto the tool or workpiece to assist spark formation and flushing, thereby improving stability and efficiency [1], [2].

Several studies have been dedicated to understanding the physical mechanisms and performance enhancement provided by UV-EDM. Zhang et al. [1] analyzed the debris removal process in ultrasonic vibration-assisted EDM drilling and reported a significant improvement in flushing efficiency. Yin et al. [2] introduced a novel longitudinal–torsional vibration system and demonstrated its superior performance in micro-hole machining. Similarly, Hirao et al. [3] found that ultrasonic vibration applied to the electrode led to reductions in tool wear and improved surface finish. Lin et al. [4] explored hybrid UV-EDM with abrasive jet machining and observed enhanced material removal and surface characteristics.

Gas-based UV-EDM has also been studied. Xu et al. [5] identified key mechanisms for material removal of cemented carbides in gas media, while Lei et al. [6] proposed laminated electrodes for microgroove machining, showing clear improvements in both groove quality and depth. In parallel, Singh et al. [7] and Shervani-Tabar et al. [15] used finite element modeling (FEM) to simulate material removal under ultrasonic assistance, reinforcing the understanding of process dynamics.

The synchronization of ultrasonic vibration with EDM pulses has been investigated by Kremer et al. [8], who observed better discharge stability and efficiency. Xing et al. [9] conducted a parameter sensitivity analysis and noted that $T_{on}$, $T_{off}$, IP, and ultrasonic amplitude are critical for micro-EDM performance. Wang et al. [10] provided detailed analysis of surface generation and removal phenomena, while Praneetpongrung et al. [11] showed improvements in the machining of ceramics like $Si_3N_4$ using UV-EDM.

The role of cavitation and hydrodynamic effects was further emphasized by Wang et al. [12], highlighting their contribution to surface formation. Lin et al. [13] integrated magnetic fields with UV-EDM and reported significant improvements in debris evacuation and surface morphology. Choubey et al. [14] compared material removal in EDM with and without ultrasonic vibration and found the latter consistently outperformed in micro-EDM applications. Hou and Bai [16] introduced a relative three-dimensional ultrasonic vibration method to further enhance debris ejection, and Zhang et al. [17] systematically studied the effect of ultrasonic amplitude on machining performance, revealing a strong correlation between amplitude and surface finish.

Although the majority of studies have focused on experimental validations, modeling efforts primarily rely on regression-based approaches such as response surface methodology (RSM). However, RSM may fail to capture the complex nonlinear behavior and noise sensitivity inherent in surface roughness formation. For example, in UV-EDM, Ra ☞

can fluctuate due to stochastic factors such as spark instabilities and cavitation collapse, which are not adequately modeled by quadratic polynomials.

To overcome these limitations, advanced machine learning-based surrogate models such as Gaussian Process Regression (GPR) offer promising alternatives. GPR has the ability to model nonlinear functions with uncertainty quantification and provides smooth interpolation between data points. This makes it particularly suitable for modeling surface roughness in processes like UV-EDM, where traditional models underperform.

Therefore, this study proposes a GPR-based modeling and optimization framework for predicting and minimizing surface roughness (Ra) in UV-EDM. The objectives are: (1) to construct a GPR model based on a five-factor Box–Behnken experimental design, (2) to evaluate and compare the prediction accuracy of GPR versus RSM, and (3) to identify optimal machining conditions for Ra minimization using the GPR model. The results are expected to contribute to more accurate and robust modeling strategies for precision surface finish in advanced EDM processes.

## 2. METHODOLOGY

### 2.1. Experimental work

In this work, experiments utilized a custom-configured ultrasonic vibration-assisted EDM system (Figure 1), with the ultrasonic vibration module functioning at a constant frequency of 20 kHz. The workpiece material was 90CrSi tool steel, and subjected to heat treatment to achieve a final hardness range of 58-60 HRC. The electrode utilized in all trials was commercially pure copper, while Diel MS 7000 dielectric fluid was employed to maintain effective flushing and thermal stability during the machining process.

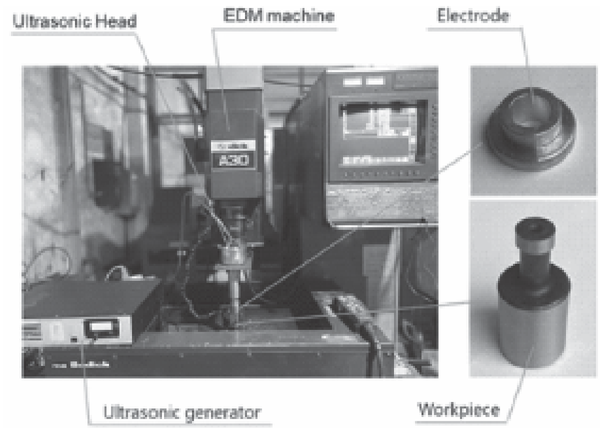


*Figure 1. Experimental setup*

To investigate the influence of process parameters on surface roughness (Ra) in ultrasonic vibration-assisted electrical discharge machining (UV-EDM), a five-factor Box–Behnken Design (BBD) was employed. BBD is a response surface methodology (RSM)-based design that is efficient for developing second-order polynomial models while avoiding extreme corner points. Five independent variables were considered in this study: Ultrasonic amplitude A (mm), pulse on-time $T_{on}$ (μs), pulse off-time $T_{off}$ (μs), discharge current IP (A), and spark voltage SV (V). Each variable was evaluated at three levels (low, medium, and high), coded as −1, 0, and +1, respectively. The design matrix consisted of 46 experimental runs, which is the standard number of runs for BBD with five variables. The experimental plan is outlined in Table 1.

*Table 1. Experimental plan and output results*

| Trial | A | $T_{on}$ | $T_{off}$ | IP | SV | Ra (µm) |
|---|---|---|---|---|---|---|
| 1 | 1.15 | 12 | 12 | 5 | 5 | 4.028 |
| 2 | 1.15 | 16 | 12 | 7 | 5 | 7.031 |
| 3 | 1.15 | 8 | 12 | 3 | 5 | 2.335 |
| 4 | 1.15 | 12 | 12 | 3 | 7 | 2.635 |
| 5 | 1.6 | 12 | 12 | 3 | 5 | 3.294 |
| 6 | 1.15 | 16 | 12 | 3 | 5 | 3.538 |
| 7 | 1.15 | 8 | 8 | 5 | 5 | 2.487 |
| 8 | 1.15 | 16 | 8 | 5 | 5 | 5.773 |
| 9 | 1.15 | 8 | 12 | 5 | 7 | 3.190 |
| 10 | 1.15 | 8 | 16 | 5 | 5 | 3.050 |
| 11 | 1.15 | 12 | 12 | 5 | 5 | 4.023 |
| 12 | 1.15 | 12 | 8 | 5 | 3 | 3.770 |
| 13 | 1.15 | 12 | 12 | 7 | 3 | 5.172 |
| 14 | 1.15 | 16 | 16 | 5 | 5 | 3.850 |
| 15 | 1.15 | 12 | 12 | 5 | 5 | 4.018 |
| 16 | 1.75 | 12 | 12 | 7 | 5 | 3.560 |
| 17 | 1.15 | 12 | 12 | 5 | 5 | 4.015 |
| 18 | 1.15 | 12 | 8 | 5 | 7 | 3.550 |
| 19 | 1.6 | 16 | 12 | 5 | 5 | 3.555 |
| 20 | 1.6 | 12 | 8 | 5 | 5 | 3.580 |
| 21 | 1.75 | 12 | 12 | 5 | 7 | 3.610 |
| 22 | 1.15 | 8 | 12 | 7 | 5 | 3.190 |
| 23 | 1.75 | 8 | 12 | 5 | 5 | 2.687 |
| 24 | 1.6 | 12 | 16 | 5 | 5 | 3.760 |
| 25 | 1.15 | 12 | 8 | 7 | 5 | 5.200 |
| 26 | 1.15 | 12 | 16 | 5 | 3 | 3.755 |
| 27 | 1.15 | 12 | 12 | 5 | 5 | 4.026 |
| 28 | 1.15 | 12 | 8 | 3 | 5 | 3.787 |
| 29 | 1.15 | 16 | 12 | 5 | 3 | 3.530 |
| 30 | 1.15 | 16 | 12 | 5 | 7 | 3.780 |
| 31 | 1.15 | 12 | 16 | 7 | 5 | 4.720 |
| 32 | 1.15 | 12 | 16 | 5 | 7 | 4.770 |
| 33 | 1.75 | 12 | 16 | 5 | 5 | 4.790 |
| 34 | 1.15 | 12 | 12 | 7 | 7 | 4.710 |
| 35 | 1.75 | 12 | 8 | 5 | 5 | 4.300 |
| 36 | 1.6 | 12 | 12 | 7 | 5 | 4.441 |
| 37 | 1.15 | 8 | 12 | 5 | 3 | 3.010 |
| 38 | 1.15 | 12 | 12 | 5 | 5 | 4.031 |
| 39 | 1.6 | 12 | 12 | 5 | 7 | 4.740 |
| 40 | 1.75 | 16 | 12 | 5 | 5 | 3.540 |
| 41 | 1.6 | 8 | 12 | 5 | 5 | 2.675 |
| 42 | 1.6 | 12 | 12 | 5 | 3 | 4.210 |
| 43 | 1.15 | 12 | 12 | 3 | 3 | 3.161 |
| 44 | 1.75 | 12 | 12 | 3 | 5 | 3.785 |
| 45 | 1.15 | 12 | 16 | 3 | 5 | 3.685 |
| 46 | 1.75 | 12 | 12 | 5 | 3 | 4.116 |

Surface roughness (Ra) was measured by using a Mitutoyo SJ-210 surface roughness tester. To account for spatial variability in surface texture, each sample was measured at three different locations on the machined surface. The final Ra value for each experiment was obtained by calculating the arithmetic mean of the three readings. The Ra values obtained from all experiments are given in the last column of Table 1.

### 2.2. Data Processing and Modeling

The experimental data were used to train two predictive models for surface roughness: a baseline second-order response surface model (RSM) and an advanced machine learning model using Gaussian Process Regression (GPR). The RSM model served as a benchmark for performance comparison, while the GPR model was developed to improve prediction accuracy in capturing nonlinear and noisy behavior of Ra in UV-EDM.

The GPR model was trained using the fitrgp function in MATLAB®, employing a constant basis function and a squared exponential kernel. Standardization of input variables was applied prior to training to ensure numerical stability and improve generalization. The performance of both models was evaluated using the coefficient of determination ($R^2$), residual analysis, and predicted-versus-actual plots.

### 2.3. Optimization strategy

To determine the optimal process parameters for minimizing surface roughness, the trained GPR model was used as a surrogate function in a single-objective optimization framework. The objective function was defined as:

$$\min_{x \in D} R_a GPR(x) \quad (1)$$

Where x = [A, $T_{on}$, $T_{off}$, IP, SV] and D denotes the feasible design space defined by the lower and upper bounds of each variable.

The optimization was performed using the fmincon solver in MATLAB® with the sequential quadratic programming (SQP) algorithm. The initial guess was chosen at the center point of the experimental domain. All decision variables were treated as continuous to enable smooth exploration of the response surface. The predicted minimum Ra value and the corresponding optimal parameter settings were recorded and compared with the experimental results for model validation.

## 3. RESULTS AND DISCUSSION

### 3.1. Model accuracy evaluation

The predictive performance of the Gaussian Process Regression (GPR) model for surface roughness (Ra) was assessed and compared with that of the second-order response surface model (RSM). The coefficient of determination ($R^2$) for the GPR model was calculated as 0.9284, while the corresponding value for RSM was 0.7832. This indicates that the GPR model captured approximately 92.84% of the variance in the experimental data, significantly outperforming the traditional RSM approach.

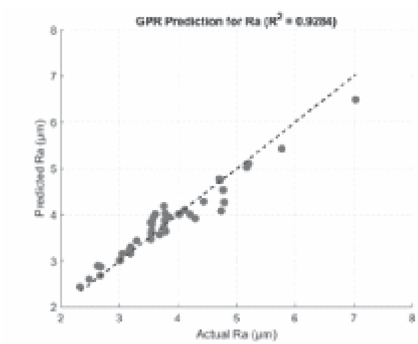


*Figure 2. Predicted vs. actual surface roughness (Ra) values from the GPR model*

Figure X shows the predicted vs. actual Ra values using the GPR model. The data points are closely clustered around the ideal line y = x, suggesting a high degree of prediction accuracy. In contrast, the RSM model exhibited a larger deviation between predicted and actual values, particularly at extreme points of the design space. These findings demonstrate that GPR is more effective in modeling nonlinear and noisy surface characteristics, which are typical in ultrasonic vibration-assisted EDM (UV-EDM) processes.

**3.2. Optimization results**

The optimal combination of process parameters for minimizing surface roughness (Ra) was determined using the trained GPR model as a surrogate objective function. The optimization was performed using MATLAB's fmincon function with bound constraints, and the objective was to minimize the predicted Ra value within the feasible design space.

The optimization yielded a minimum predicted Ra of 2.278 µm at the following combination of process parameters: A = 1.23 (µm), $T_{on}$ = 8.0 (µs), $T_{off}$ = 10.8(µs), IP = 4.0 (A), SV = 5.05 (V).

These results indicate that a relatively high amplitude of ultrasonic vibration, short pulse on-time, and low-to-moderate levels of discharge current and voltage are beneficial for achieving minimum surface roughness in ultrasonic vibration-assisted EDM.

Notably, the optimal pulse on-time ($T_{on}$) is located at the lower bound of the design space, reaffirming previous findings that longer pulse durations tend to degrade surface finish. The predicted Ra value of 2.278 µm demonstrates the ability of the GPR model to accurately represent the nonlinear response of Ra and guide the selection of process parameters with high precision.

The optimized solution lies well within the experimental region defined by the Box-Behnken design, ensuring that the result is practically feasible and experimentally verifiable. This confirms the applicability and reliability of the GPR-based optimization framework for single-objective surface quality enhancement in UV-EDM

To assess the reliability of the model, an experiment was carried out using the optimal parameter set. The measured surface roughness of the machined part was Ra = 2.426 µm. Compared to the predicted optimal roughness value, the measurement exhibited a deviation of 6.5% (Table 2). This small error indicates that the discrepancy between the experimentally obtained surface roughness and the predicted value is negligible. Therefore, the optimal parameter set is validated for practical application in achieving minimal surface roughness in pulse machining.

*Table 2. Experimental testing results corresponding optimal condition*

| **Output parameter** | **Predict value** | **Experiment testing value** | **Error (%)** |
|---|---|---|---|
| Roughness Ra (µm) | 2.278 | 2.426 | 6.5 |

**4. CONCLUSION**

In this study, a Gaussian Process Regression (GPR) model was developed and applied to predict and minimize surface roughness (Ra) in ultrasonic vibration-assisted electrical discharge machining (UV-EDM). A ☞

Box-Behnken experimental design with five input variables ultrasonic amplitude (A), pulse on-time ($T_{on}$), pulse off-time ($T_{off}$), discharge current (IP), and spark voltage (SV) was utilized to train and validate the model. The main conclusions can be summarized as follows:

1) The GPR model significantly outperformed the traditional response surface model (RSM) in terms of prediction accuracy for Ra. The GPR model achieved an R-squared value of 0.9284, while the RSM model yielded only 0.7832.

2) The GPR-based optimization framework successfully identified an optimal set of process parameters that minimized Ra to 2.278 μm. The optimal condition corresponds to A = 1.23 (μm), $T_{on}$ = 8.0 (μs), $T_{off}$ = 10.8(μs), IP = 4.0 (A), SV = 5.05(V).

3) Residual and uncertainty analysis confirmed the robustness of the GPR model, showing low residual dispersion and higher prediction confidence near the center of the design space. This reinforces the model's suitability for practical optimization in UV-EDM.

Overall, the findings demonstrate that GPR is a highly effective and reliable modeling approach for complex and nonlinear surface responses such as Ra in UV-EDM. The proposed framework not only improves prediction fidelity but also enables data-driven optimization with uncertainty quantification, making it a valuable tool for intelligent process control and precision surface engineering.

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