

DYNAMIC RESPONSE OF A COMPOSITE WING-SHAPED PLATE UNDER IMPACT LOADING

ĐÁP ỨNG CỦA TẤM COMPOSITE DẠNG CÁNH MÁY BAY DƯỚI TÁC ĐỘNG CỦA TẢI TRỌNG VÀ ĐẬP

Bui Van Binh

Electric Power University

ABSTRACT

This paper presents an analysis of the response of a composite wing-shaped plate subjected to impact loading. The study investigates the effects of geometric configurations, material properties and impact location on the transient deflection response. The computational model is based on Mindlin's plate theory. A numerical simulation program is developed in Matlab, employing an eight-node isoparametric plate element with five degrees of freedom per node.

Keywords: *Corrugated core; Sandwich plate; Transient responses; Mindlin plate theory.*

TÓM TẮT

Bài báo trình bày phân tích đáp ứng của tấm composite dạng cánh máy bay dưới tác động của tải trọng va đập. Trong đó, ảnh hưởng của hình dạng hình học, vật liệu và vị trí lực tác động đến đáp ứng tức thời của độ võng được khảo sát. Mô hình tính toán dựa trên lý thuyết tấm của Mindlin. Chương trình tính toán số được xây dựng trên Matlab sử dụng phần tử tấm đẳng tham số 8 (tám) nút, mỗi nút có 5 (năm) bậc tự do.

Từ khóa: *Tấm composite dạng cánh máy bay; Đáp ứng tức thời; Lý thuyết tấm của Mindlin; Tải trọng va đập.*

1. INTRODUCTION

Fiber-reinforced composite plates are among the most advanced materials widely applied across various engineering fields due to their high strength-to-weight ratio, excellent corrosion resistance, and superior mechanical performance. In the aerospace industry, these composite plates play a critical role in the fabrication of structural components such as aircraft wings, fuselage, and interior panels, contributing to overall weight reduction, fuel

efficiency, and enhanced flight performance. Typical composites, such as carbon fiber-reinforced epoxy, enable optimal directional design due to their anisotropic nature, while also exhibiting high dynamic load resistance under harsh environmental conditions. In general engineering applications, fiber-reinforced composite plates are utilized in components such as equipment casings, mechanical structures, ship hulls, automotive bodies, sports equipment, and wind energy systems. Their flexible design capabilities, multifunctional 

integration (e.g., electrical insulation, thermal conductivity, fire resistance), and long service life make them an efficient and sustainable material solution for modern structural systems. Pandey et al. [1] presented a nonlinear transient analysis of porous functionally graded material (FGM) plates subjected to step blast loading, employing a C0 finite element formulation. The Newmark-Beta integration scheme, combined with a modified Newton-Raphson iterative method, was utilized to solve the resulting governing differential equations. The study investigates the influence of key parameters, including the power-law index, porosity coefficient, and side-to-thickness ratio, on the transient response of FGM plates under step blast loading. Wang et al. [2] developed a theoretical framework based on orthotropic thin plate theory and modal analysis to investigate the effects of peak pressure and loading duration on the structural behavior and dynamic response of a simply supported flat plate subjected to various impulse loadings. Numerical simulations of structural deflections were conducted using the explicit finite element method implemented in LS-DYNA version 971. Zhu et al. [3] employed the Membrane Factor Method and Saturation Analysis to predict the dynamic inelastic response of rectangular plates subjected to long-duration pressure pulse loading. The study discusses the influence of the adopted yield criterion and the transient response phase on the saturation behavior of rectangular plates under such loading conditions. Ma et al. [4] employed the mode superposition technique combined with experimental methods to investigate the transient response of a cantilevered plate subjected to impact loading at various locations across the plate structure. Ehrlich et al. [5] conducted an experimental investigation on the behavior of laminated composite specimens

with elliptic curvature. The experimental results indicated that, under low-velocity impact loading, the size of the specimen or structure is a critical parameter governing the dynamic response. Ji et al. [6], based on the first-order shear deformation plate theory, conducted forward and inverse analyses to investigate the free vibration and dynamic response of a cantilevered rectangular plate subjected to normal and oblique impact loadings. The numerical results for displacement and strain were compared with experimental data for plates made of isotropic materials, confirming the accuracy of the mode superposition technique using a theoretical transfer function in the inverse analysis. Gulizzi et al. [7] employed a high-order discontinuous Galerkin method to perform free-vibration analysis and assess the dynamic behavior of laminated composite plates and shells, aiming to overcome the limitations of analytical methods and the high cost of experimental testing. Malekzadeh et al. [8] employed the three-dimensional layerwise differential quadrature method in conjunction with a multi-step time integration scheme based on non-uniform rational B-splines to analyze the transient response of aerospace-grade composite sandwich panels subjected to dynamic loading. The numerical results investigated the influence of panel thickness on the deflection and stress responses under specific boundary conditions.

In Vietnam, research on wing-shaped laminated composite plate structures with stiffening elements remains relatively scarce, especially in the context of transient response analysis under impact loading. This study presents numerical investigations into effects of geometric configurations, material properties, and impact positions on the transient displacement responses of wing-shaped

laminated composite plates reinforced with box-type stiffeners. The transient displacement responses were computed using the Newmark time integration algorithm.

2. FORMULATION OF THE PROBLEM

2.1. The strain-displacement relations

According to the Reissner-Mindlin plate theory [9], the displacements (u , v , w) are referred to those of the mid-plane (u_0 , v_0 , w_0) as:

$$\begin{Bmatrix} u \\ v \\ w \end{Bmatrix} = \begin{Bmatrix} u_0 + z\theta_x \\ v_0 + z\theta_y \\ w_0 \end{Bmatrix} \text{ với } \begin{Bmatrix} \theta_x \\ \theta_y \end{Bmatrix} = \begin{Bmatrix} \frac{\partial w}{\partial x} + \phi_x \\ \frac{\partial w}{\partial y} + \phi_y \end{Bmatrix} \quad (1)$$

Here, θ_x and θ_y are the total rotations, ϕ_x and ϕ_y are the constant average shear deformations about the y and x -axes, respectively.

The z -axis is normal to the xy -plane that coincides with the mid-plane of the laminate positive downward and clockwise with x and y . The strain-displacement relations can be taken as:

$$\begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \varepsilon_{zz} \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix} = \begin{Bmatrix} \frac{\partial u_0}{\partial x} + z \frac{\partial \theta_x}{\partial x} \\ \frac{\partial v_0}{\partial y} + z \frac{\partial \theta_y}{\partial y} \\ 0 \\ \left(\frac{\partial u_0}{\partial y} + \frac{\partial v_0}{\partial x} \right) + z \left(\frac{\partial \theta_x}{\partial y} + \frac{\partial \theta_y}{\partial x} \right) \\ \frac{\partial w_0}{\partial y} + \theta_y \\ \frac{\partial w_0}{\partial x} + \theta_x \end{Bmatrix} \quad (2)$$

2.2. Finite element formulations

In laminated plate theories, the membrane $\{N\}$, bending moment $\{M\}$ and shear stress $\{Q\}$ resultants can be obtained by integration of stresses over the laminate thickness. The stress resultants-strain relations can be expressed in the form:

$$\begin{Bmatrix} \{N\} \\ \{M\} \\ \{Q\} \end{Bmatrix} = \begin{bmatrix} [A_{ij}] & [B_{ij}] & [0] \\ [B_{ij}] & [D_{ij}] & [0] \\ [0] & [0] & [F_{ij}] \end{bmatrix} \begin{Bmatrix} \{\varepsilon^0\} \\ \{\kappa\} \\ \{\gamma^0\} \end{Bmatrix} \quad (3)$$

Where:

$$([A_{ij}], [B_{ij}], [D_{ij}]) = \sum_{k=1}^n \int_{h_{k-1}}^{h_k} ([Q'_{ij}]_k) (1, z, z^2) dz \quad (i, j = 1, 2, 6) \quad (4)$$

$$[F_{ij}] = \sum_{k=1}^n f \int_{h_{k-1}}^{h_k} ([C'_{ij}]_k) dz; \quad f = 5/6; \quad i, j = 4, 5 \quad (5)$$

n : number of layers, h_{k-1} , h_k : the position of the top and bottom faces of the k^{th} layer.

$[Q'_{ij}]_k$ and $[C'_{ij}]_k$: reduced stiffness matrices of the k^{th} layer (see [9]).

In the present work, eight noded isoparametric quadrilateral element with five degrees of freedom per nodes is used. The displacement field of any point on the mid-plane given by:

$$u_0 = \sum_{i=1}^8 N_i(\xi, \eta) \cdot u_i; \quad v_0 = \sum_{i=1}^8 N_i(\xi, \eta) \cdot v_i; \\ w_0 = \sum_{i=1}^8 N_i(\xi, \eta) \cdot w_i;$$



$$\theta_x = \sum_{i=1}^8 N_i(\xi, \eta) \cdot \theta_{xi}; \quad \theta_y = \sum_{i=1}^8 N_i(\xi, \eta) \cdot \theta_{yi} \quad (6)$$

Where: $N_i(\xi, \eta)$ are the shape function associated with node i in terms of natural coordinates (ξ, η) .

For free and forced vibration analysis, the damping effect is neglected; the governing equations are [9]:

$$[M]\{\ddot{u}\} + [K]\{u\} = \{0\} \text{ or } \{[M] - \omega^2[K]\}\{u\} = \{0\} \quad (7)$$

$$\text{And } [M]\{\ddot{u}\} + [K]\{u\} = f(t) \quad (8)$$

In which $\{u\}$, $\{\ddot{u}\}$ are the global vectors of unknown nodal displacement, acceleration, respectively. $[M]$, $[K]$, $f(t)$ are the global mass matrix, stiffness matrix, applied load vectors, respectively.

Where:

$$[M] = \sum_1^n [m^*]_e; \quad [K] = \sum_1^n [k^*]_e; \quad \{f(t)\} = \sum_1^n \{f_e^*(t)\} \quad (9)$$

$$\text{In which: } [m^*]_e = [T]^T [m]_e [T];$$

$$[k^*]_e = [T]^T [k]_e [T]; \quad [f^*]_e = [T]^T \{f\}_e$$

With n is the number of element; $[T]$ is the transformation matrix.

3. NUMERICAL RESULTS

3.1. The composite wing-shaped plate

Consider a composite wing-shaped

plate is shown in Figure 1; the dimensions of $W = 0.8$ (m), $l_i = 0.15$ (m), and $a = b = c = 0.045$ (m); total thickness $t = 7.25$ (mm), tapered angles of $\varphi = 30^\circ, 40^\circ, 45^\circ$. Lamination schemes: five laminated configurations $[\theta^\circ/-\theta^\circ/\theta^\circ/-\theta^\circ/\theta^\circ]$ are constructed.

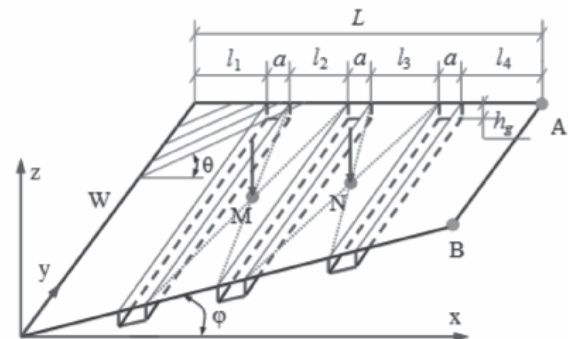


Figure 1. The composite wing-shaped plate with three box-type stiffeners

The boundary condition: clamped at $x = 0$.

Table1. Woven fabric composite material properties (MAT 1)

E_1, E_2 (GPa)	E_3 (GPa)	$G_{12, 13, 23}$ (GPa)	ν_{12}	ν_{13}	ν_{23}	ρ (kg/m ³)
63.8	11.0	4.2	0.064	0.064	0.32	1523

In the transient analysis, the same folded composite plates subjected to an exploded knife-edge loading condition scheme of $q = 1$ kN/m, which having $t_1 = 1$ ms, $t_2 = 2$ ms, $t_3 = 50$ ms, illustrated in Figure 2.

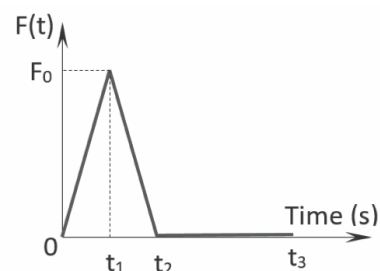


Figure 2. Impact loading scheme

3.2. Effects of the geometric configurations

To investigate the effects of geometric configuration on the transient displacement response of point A, the laminated wing-shaped plates with the stacking sequences $[45^\circ/-45^\circ/45^\circ/-45^\circ/45^\circ]$ are considered for three different taper angles $\varphi = 30^\circ, 40^\circ, 45^\circ$ subjected impact loading at point N. The material properties shown in Table 1; The results are compared for various cases as illustrated in Figure 3.

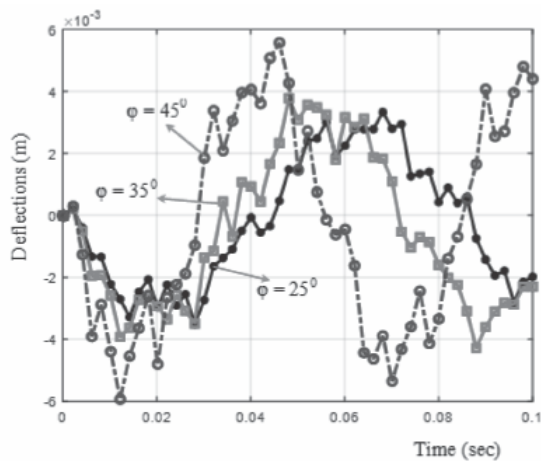


Figure 3. Effects of taper angles on transient displacement responses

Figure 3 illustrates the variation in deflection response of composite plates subjected to an instantaneous impact load as the taper angle φ changes. It is observed that as φ increases from 25° to 45° , the vibration amplitude increases significantly, indicating a reduction in the overall stiffness of the structure. The response corresponding to $\varphi = 45^\circ$ not only exhibits a higher amplitude but also demonstrates more intense oscillations compared to the other two cases. This leads to a considerable rise in local stress and strain, thereby increasing the risk of interlaminar delamination, particularly near the edges

or impact zones. These findings highlight the critical role of taper angle selection in controlling the dynamic behavior of laminated composite structures under impact loading.

3.3. Effects of impact locations

To investigate the effects of impact location on the transient displacement response, the composite plate with a taper angle of $\varphi = 45^\circ$ is subjected to impact loading at two distinct locations, namely points M and N, as illustrated in Figure 1. The corresponding displacement responses (at point A) are compared for both impact locations, with the results presented in Figure 4.

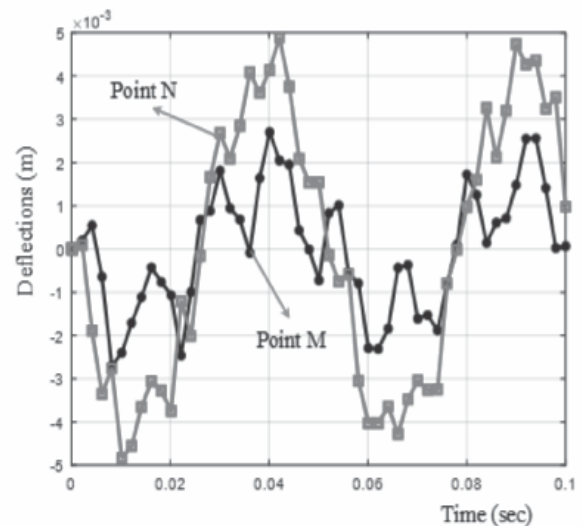


Figure 4. Effects of impact location on transient displacement responses

The results indicate that the impact at point N generates a significantly higher deflection amplitude compared to point M. These findings highlight the sensitivity of composite structures to local impact positions and emphasize the need for careful consideration of potential impact zones during the design process.

3.4. Effects of material properties

To investigate the effects of material properties on the transient displacement response, laminated composite plates composed of three different materials detailed in Tables 1, 2, and 3 are considered. All plates share the same stacking sequence of $[45^\circ/-45^\circ/45^\circ/-45^\circ]$ and a taper angle of $\varphi = 45^\circ$, and are subjected to impact loading at point N. The corresponding responses are compared between two representative cases, as illustrated in Figure 5.

Table 2. Carbon fiber T700 material properties (MAT 2)

E_1 , (GPa)	E_2, E_3 (GPa)	$G_{12, 13}$ (GPa)	G_{23} (GPa)	$\nu_{12, 13}$	ν_{23}	ρ (kg/ m ³)
165	15	6.4	5.5	0.286	0.366	1611

Table 3. Graphite/epoxy AS4/3502 material properties (MAT 3)

E_1 (GPa)	E_2, E_3 (GPa)	G_{12} (GPa)	$G_{13, 23}$ (GPa)	ν_{13}	$\nu_{12, 23}$	ρ (kg/ m ³)
131	9.12	7.1	5.92	0.24	0.34	1600

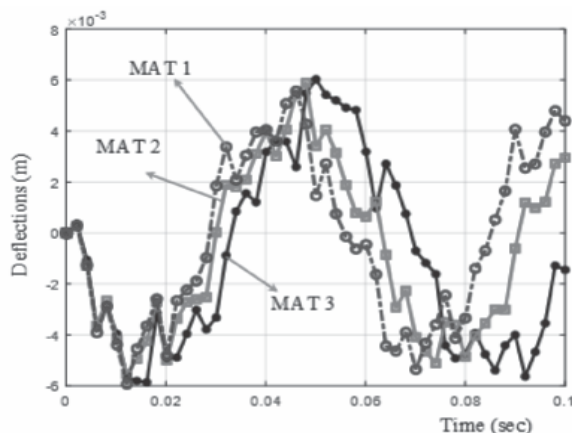


Figure 5. Effects of material properties on transient displacement responses

The results clearly demonstrate that MAT 1 exhibits the largest deflection amplitude, suggesting lower structural stiffness and reduced resistance to impact-induced deformation. In contrast, MAT 3 shows the smallest amplitude and the most stable response, indicating superior stiffness and dynamic performance. These findings underscore the importance of material selection in enhancing the impact resistance and durability of laminated composite structures.

4. CONCLUSIONS

This study presents a numerical investigation into the transient displacement response of a wing-shaped laminated composite plate subjected to impact loading. The effects of geometric configuration, impact location, and material properties on the structural response were systematically examined using a finite element model based on Mindlin plate theory. The results show that increasing the taper angle leads to a significant rise in vibration amplitude, indicating a reduction in overall stiffness and an increased risk of interlaminar delamination under impact. Additionally, the location of the impact plays a crucial role, as impacts occurring near sensitive regions result in higher local deformations. Material selection was also found to strongly influence the structural behavior: plates made from stiffer composite materials exhibited lower displacement amplitudes and more stable responses. These findings emphasize the importance of optimizing both geometry and material choice during the design of composite structures to improve their impact resistance and dynamic performance. The outcomes of this research contribute valuable insights for the development of lightweight, high-performance components in aerospace and mechanical engineering applications where transient loads are critical design considerations.

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