APPLICATION OF SAW TECHNIQUE FOR MULTI-OBJECTIVE OPTIMIZATION OF A TWO-STAGE HELICAL GEARBOX TO ENHANCE EFFICIENCY AND REDUCE LENGTH

ỨNG DỤNG PHƯƠNG PHÁP SAW ĐỂ GIẢI BÀI TOÁN TỐI ƯU ĐA MỤC TIÊU CHO HỘP GIẢM TỐC BÁNH RĂNG TRỤ HAI CẤP NHẰM NÂNG CAO HIỆU SUẤT VÀ GIẢM CHIỀU DÀI

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ABSTRACT

This paper demonstrates the application of the Simple Additive Weighting (SAW) approach to solve the Multi-Objective Optimization Problem (MOOP) related to a two-stage helical gearbox. The goal is to determine the most effective primary design factors that will improve gearbox efficiency and reduce gearbox length. This study specifically chose three crucial design parameters: the gear ratio of the first stage, and the coefficients of wheel face width (CWFW) for both the first and second stages. Furthermore, the inten-tional choice of the SAW technique as a Multi-Criteria Decision Making (MCDM) method was made in order to tackle the difficulty of multi-objective optimization. In addition, the Multi-Expert Ranking Evaluation with Compensation (MEREC) technique was selected for calculating the weight criterion. The study's findings aid in determining optimal values for three important design parameters in the creation of a two-stage helical gearbox.

Keywords: *Multi-objective optimization; Helical gearbox; Gearbox efficiency; Gearbox length; SAW method; MEREC method.*

TÓM TẮT

Bài báo này trình bày ứng dụng phương pháp Simple Additive Weighting (SAW) để giải bài toán tối ưu hóa đa mục tiêu hộp giảm tốc bánh răng trụ hai cấp. Mục tiêu của nghiên cứu là xác định các thông số thiết kế chính tối ưu để nâng cao hiệu suất và giảm chiều dài hộp. Trong nghiên cứu này, ba thông số thiết kế chính gồm tỉ số truyền cấp nhanh và hệ số chiều rộng răng của cả cấp nhanh và cấp chậm. Bên cạnh đó, phương pháp SAW được chọn để giải bài toán tối ưu hóa đa mục tiêu. Ngoài ra, phương pháp MEREC đã được chọn để tính toán trọng số của các tiêu chí. Các kết quả của nghiên cứu này giúp xác định các giá trị tối ưu của ba thông số thiết kế chính khi thiết kế hộp giảm tốc bánh răng trụ hai cấp.

Từ khóa: *Tối ưu hóa đa mục tiêu; Hộp giảm tốc bánh răng trụ; Hiệu suất hộp giảm tốc; Chiều dài hộp giảm tốc; Phương pháp SAW; Phương pháp MEREC.*

1. INTRODUCTION

The gearbox is said to be the most important part of the mechanical transmission drive system. Reducing the torque and speed transferred from the working shaft to the motor shaft is its main goal. As a result, gearbox optimization has drawn attention from scientists.

A. Parmar et al. [1] performed an optimization analysis of a planetary gearbox using the NSGA-II (Non-Dominated Sorting Genetic Algorithm II) technique, taking into consideration significant regular mechanical and tribological restrictions. Applying the study's recommendations resulted in a significant reduction of weight and power loss when comparing the outcomes of single-objective optimization with and without tribological limitations. M. Patil et al. [2] presented a novel multi-objective optimization of a two-stage spur gearbox using the NSGA-II method. This attempt involves two aim functions: minimum gearbox power losses and minimum gearbox volume. The results of the study indicate that there is a high probability of wear failure in solutions that are obtained using single objective minimization. Furthermore, the total power loss is reduced by half when multi-goal optimization is used instead of single objective optimization. Kalyanmoy D. and Sachin J. [3] used the NSGA-II approach in their work. The investigation revealed that a larger module is required for a greater provided power in order to achieve the same output speed requirement. Additionally, wear stress failure is more important than bending stress failure for gearboxes with low power, while the inverse is true for gearboxes with high power. R. C. Sanghvi et al. [4] used three methodologies to tackle the multi-objective optimization of a two-stage helical gear train: NSGA-II, genetic algorithms (GA), and the Matlab optimization toolbox. Two of the objectives of the study are the optimization of volume and load carrying capacity. The comparison of the findings shows that the NSGA-II strategy produced superior outcomes than the other methods for both objectives.

The multi-objective design of transmission utilizing helical gear pairs is investigated by Sabarinath P. et al. [5]. The aim functions are indicated by the gear volumes and the opposing number of overlap ratio. The optimization problem was solved by the study using the Parameter Adaptive Harmony Search Algorithm (PAHS). A multi-objective optimization problem using two-stage helical gear sets in a helical hydraulic rotary actuator is presented by Liu S. et al. [6]. Utilizing four variations of the three objectives: volume, transmission efficiency, and maximum contact stress, the optimization is carried out using the NSGA-II approach. The results imply that in the optimum zone, there is a trade-off between volume and maximum contact stress. Additionally, considering different combinations improves the longevity of the optimization results. Lagresle et al. [7] solved the optimization of tooth modifications for spur and helical gears using a mono-objective self-adaptive algorithm. The technique is based on the particle swarm optimization (PSO) technology. The maximal contact pressures and the root mean square values of the transmission error signal have been improved by the multi-objective optimization. Chrystopher V.T. et al. [8] used the MCDM approach to find the best gear material for a gearbox in order to reduce surface fatigue and increase wear resistance. The aim of this research is to enhance the effectiveness of surface fatigue resistance in gearbox applications. ☞

J. A. Korta et al. carried out a multi-objective micro-geometry optimization of the gear tooth using response surface techniques [9]. Three objectives were taken into consideration when Edmund S. M. and Rajesh A. [10] solved a multi-objective op-timization using the NSGA-II and the TOPSIS (Technique for Order of Preference by Similarity to Ideal Solution) technique. The three objectives were the gearbox volume, the power output, and the center distance. The results of the investigation shed light on tiny gearbox design. A study to jointly optimize a gearbox and an electric motor for the purpose of designing an electric vehicle drive system was presented in [11]. The work's goal functions are to minimize the drive system's weight and overall energy loss. The optimization outcomes are contrasted with earlier findings to highlight collaborative optimization's further potential. It has been observed that when the drive system is optimized overall, increasing the gear ratio enhances the system's overall efficiency.

An ideal multi-objective analysis of a cycloid pin gear planetary reducer was given by Wang, Y. et al. [12]. The study concentrated on applying Pareto optimal solutions to analyze the reducer volume, turning arm bearing force, and pin maximum bending stress in order to minimize each of these three goals. The results of the study show that the modified algorithm can produce Pareto optimum solutions that are more efficient than those produced by the standard design. Multi-objective optimization was studied by Chong, T.H. et al. [13] in order to minimize vibration in cylindrical gear pairs induced by meshing between gears and reduce gear size. Strength and geometric constraints are taken into consideration in the design process to reduce the size and vibration of cylindrical gear pairs. The results of Park C.I.'s research on a multi-objective optimization problem for helical gear noise reduction were published [14]. The response surface method and experiment design were used in the study to solve the multi-objective optimization problem. An approach to spur gear set design optimization was proposed by Jawaz Alam & Sumanta Panda [15]. Its goals are to minimize gear weight, contact stress throughout the contact path, and appropriate film thickness at the contact point. For a profile-modified spur gear set with adequate film thickness at the point of contact, this work used particle swarm optimization, particle swarm optimization based teaching learning optimization, and Jaya approaches to guarantee a significant decrease in weight and contact stress. The study's findings demonstrate that, as compared to conventional designs, the gear design with optimal addendum coefficient values inside the design area performs noticeably better.

This research utilizes SAW strategy to address the Multiple Criteria Decision Making (MCDM) problem. Additionally, the study applies MEREC method to determine the weights of the criteria for solving MOOP of a two-stage helical gearbox. The goal is to reduce the dimensions of the gearbox and improve its efficiency. The investigation yielded findings that led to a list of the most optimal key design factors for the gearbox.

2. OPTIMIZATION PROBLEM

2.1. Determining gearbox length

For a two stage helical gearbox, the length L_{gb} can be calculated by (Figure 1):

$$L = d_{w11} + d_{w21}/2 + d_{w12}/2 + d_{w22} + 20 \quad (1)$$

Where, d_{w1i}, d_{w2i} (i=1÷2) denote the pitch diameter of the pinion and the gear of stage 1 and 2 which are found by:

$$d_{w1i} = 2 \cdot a_{wi} / (u_i + 1) \quad (2)$$

$$d_{w2i} = 2 \cdot a_{wi} \cdot u_i/(u_i + 1) \quad (3)$$

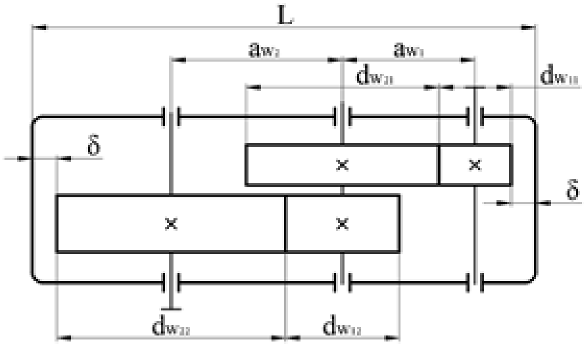

Figure 1. Calculated schema

In Equations (2) and (3), X_{bai} and a_{wi} (i=1÷2) are the CWFW and the center of the gearbox of stage i; a_{wi} is determined by [1]:

$$a_{wi} = k_a \cdot (u_i + 1) \cdot \sqrt[3]{T_{1i} \cdot k_{H\beta}/([AS_i]^2 \cdot u_i \cdot X_{bai})} \quad (4)$$

In (4), T_{1i} (i=1÷2) is the pinion torque of stage i which can be found by:

$$T_{1i} = \frac{T_r}{\prod_{j=i}^{3}(u_i \cdot \eta_{hg}^{3-i} \cdot \eta_{be}^{4-i})} \quad (5)$$

2.2. Determining gearbox efficiency

The efficiency of the gearbox (%) is determined by:

$$\eta_{gb} = 100 - \frac{100 \cdot P_l}{P_{in}} \quad (6)$$

In (6), P_l is the total power loss in the gearbox which can be found by [2]:

$$P_l = P_{lg} + P_{lb} + P_{ls} + P_{Z0} \quad (7)$$

Where, P_{lg}, P_{lb}, P_{ls}, and P_{Z0} denote the power loss in the gearings, in bearings, in seals, and in the idle motion which are determined as in [3].

2.3. Objective functions and constrains

2.3.1. Objectives functions

The MOOP in this work has two single objectives:

- Minimizing the gearbox length:

$$\min f_1(X) = L_{gb} \quad (8)$$

- Maximizing the gearbox efficiency:

$$\min f_2(X) = \eta_{gb} \quad (9)$$

Consider X to be the vector that reflects the design parameters. A two-stage helical gearbox is defined by five main design factors: u_1, X_{ba1}, X_{ba2}, AS_1, and AS_2 [3]. Moreover, research has demonstrated that the optimal values for AS_1 and AS_2 correspond to their maximum values. Thus, three key design aspects, namely u_1, Xb_{a1}, and Xb_{a2}, were chosen as variables. Consequently, we now possess:

$$X = \{u_1, X_{ba1}, X_{ba2}\} \quad (10)$$

2.3.2. Constrains

For stage i[th] (i=1÷2) of a two stage helical gearbox, u_i = 1÷9; X_{bai} = 0.25÷0.4 [3], the MOOP requires the following constraints:

$$1 \le u_1 \le 9 \text{ and } 1 \le u_2 \le 9 \quad (11)$$

$$0.25 \le Xba_1 \le 0.4 \text{ and } 0.25 \le Xba_2 \le 0.4 \quad (12)$$

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3. METHODOLOGY

3.1. Method for solving MOOP

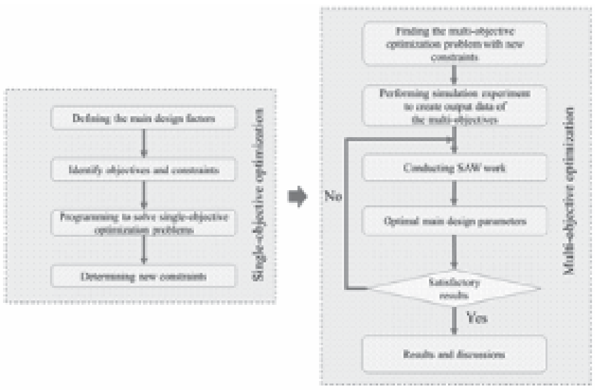


Figure 2. Procedure to solve the MOOP

Table 1. Input variables

Variable	Symbol	Lower limit	Upper limit
Gear ratio of first stage	u_1	1	9
CWFW of stage 1	X_{ba1}	0.25	0.4
CWFW of stage 2	X_{ba2}	0.25	0.4

The work aims to achieve two objectives: minimizing the length of the gearbox and maximizing its efficiency. Table 1 presents the three essential design parameters that serve as input variables for the MOOP. Furthermore, the approach outlined in reference [3] was employed to address the multi-objective optimization problem (MOOP). The procedure for carrying out this function is illustrated in Figure 2. The strategy consists of two distinct phases: The initial phase is dedicated to resolving the problem of single-objective optimization by aiming to reduce the disparities among the input components, as specified in Table 1. In the second stage, the focus will be on resolving the MOOP issue by identifying the key design aspects that yield the best results. If the levels of the variables in the second stage are not sufficiently close to one other (less than 0.02), the SAW issue will be reassessed using the smaller difference between two levels of the input factors.

3.2. Method to solve MCDM

The SAW technique was proposed in 2006 [3]. The steps to use this method are as follows:

- Making the initial decision-making matrix:

$$X = \begin{matrix} \\ A_1 \\ A_2 \\ \cdots \\ A_m \end{matrix} \begin{matrix} C_1 & C_2 & \cdots & C_n \\ \left[\begin{matrix} y_{11} \\ y_{21} \\ \cdots \\ y_{m1} \end{matrix}\right. & \begin{matrix} y_{12} \\ xy_{22} \\ \cdots \\ y_{m2} \end{matrix} & \begin{matrix} \cdots \\ \cdots \\ \cdots \\ \cdots \end{matrix} & \left.\begin{matrix} y_{1n} \\ y_{2n} \\ \cdots \\ y_{mn} \end{matrix}\right] \end{matrix} \quad (13)$$

In (13), m and n are the numbers of options and criteria.

- Calculating the normalized matrix by:

$$n_{ij} = \frac{r_{ij}}{\max r_{ij}} \quad (14)$$

$$n_{ij} = \frac{\min r_{ij}}{r_{ij}} \quad (15)$$

Equation (14) is used for the gearbox efficiency and (15) for the gearbox length.

- Finding the preference value for each option:

$$V_i = \sum_{j=1}^{n} w_j \cdot n_{ij} \quad (16)$$

- Sort the options in order according on the notion that the option with the highest V_i is considered the most beneficial.

3.3. Method to calculate criterion weights

In this work, the MEREC technique was applied to find the criteria weights. The following steps are need to conduct for using this method [4]:

+) Constructing the initial matrix using the identical process employed in the initial stage of the SAW method.

+) After normalizing the matrix, find the values of its elements by:

- For gearbox efficiency objective:

$$h_{ij} = \frac{minx_{ij}}{x_{ij}} \quad (17)$$

- For gearbox length objective:

$$h_{ij} = \frac{x_{ij}}{maxx_{ij}} \quad (18)$$

+) Finding the effectiveness of the options S_i by:

$$S_i = ln\left[1 + \left(\frac{1}{n}\sum_j \left|ln(h_{ij})\right|\right)\right] \quad (19)$$

+) Determining the efficiency of the i^{th} option S'_{ij} by:

$$S'_{ij} = Ln\left[1 + \left(\frac{1}{n}\sum_{k,k\neq j} \left|ln(h_{ij})\right|\right)\right] \quad (20)$$

+) Calculating the removal effect of the j^{th} criterion E_j:

$$E_j = \sum_i \left|S'_{ij} - S_i\right| \quad (21)$$

+) Determining the criteria's weight by:

$$w_j = \frac{E_j}{\sum_k E_k} \quad (22)$$

4. SINGLE-OBJECTIVE OPTIMIZATION

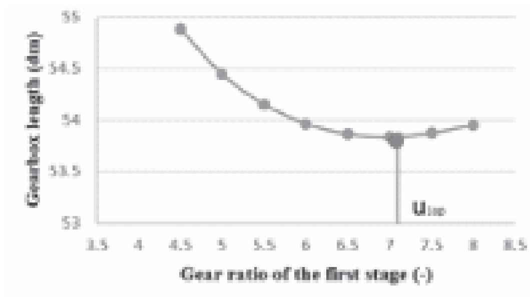


Figure 3. Relation between u_1 and L_{gb}

In this work, the direct search approach was applied to address the issue of optimizing a single objective. Furthermore, a Matlab program was employed to analyze two distinct single-objective issues: enhancing gearbox efficiency and minimizing L_{gb}. The program's findings are succinctly elucidated in the subsequent observations: Figure 3 depicts the association between the variables u_1 and L_{gb}. The ideal value of u_1 is determined in order to obtain the smallest value of L_{gb}, as shown in Figure 3. Figure 4 illustrates the correlation between η_{gb} and u_1. From the Figure, it is clear that there is a specific value of u_1 where η_{gb} reaches its maximum value, indicating an optimal value. The relationships between X_{ba1} and X_{ba2}, as well as L_{gb} and η_{gb}, are depicted in Figures 5 and 6, respectively. An enlargement of X_{ba1} and X_{ba2} leads to a reduction in both L_{gb} and η_{gb}. Furthermore, Figure 7 illustrates the relationship between the best values for u_1 and the u_t. The optimal values of u_1 were used to ☞

establish the revised constraints for the variable u_1, as shown in Table 2.

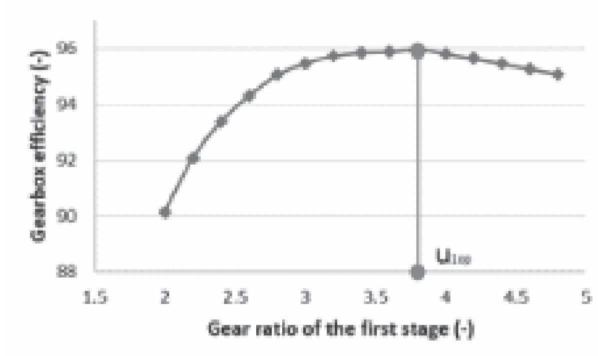
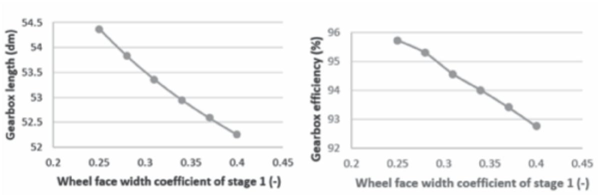
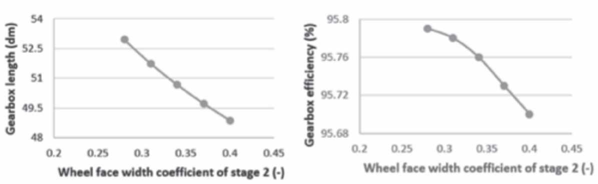


Figure 4. Relation between u_1 and η_{gb}



a) b)

Figure 5. Relation between X_{ba1} and L_{gb} (a) and η_{gb} (b)



a) b)

Figure 6. Relation between X_{ba2} and L_{gb} (a) and η_{gb} (b)

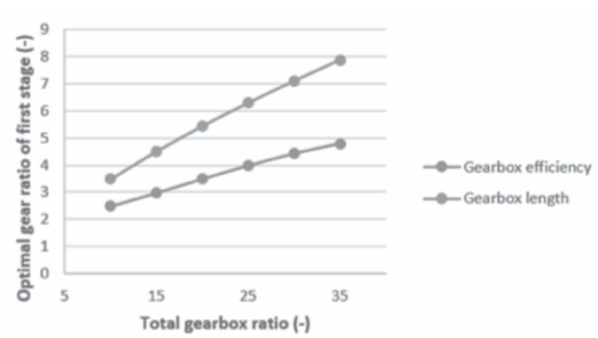


Figure 7. Relation between ugb and optimal values of u_1

Table 2. New constraints of u_1

u_{gb}	u_1	
	Lower limit	Upper limit
10	2.39	3.60
15	2.88	4.61
20	3.39	5.54
25	3.88	6.39
30	4.32	7.19
35	4.69	7.86

5. MULTI-OBJECTIVE OPTIMIZATION

A software application for the computer has been created to carry out simulation experiments. The experiment examined the values of u_{gb}, which were 10, 15, 20, 25, 30, and 35. Here are the solutions for the issue with u_{gb} = 20. The gearbox ratio listed above was used for 125 initial testing iterations, as described in Section 3. The experiment will yield output data on the gearbox length's efficiency, which will be conveyed to SAW as input parameters for solving the MOOP. This procedure will continue until the difference between two levels of u_1 is smaller than 0.02. Table 3 presents the primary design parameters and output responses for the fifth and final iteration of the SAW experiment, with a u_{gb} value of 20. The criterion weights were determined using the MEREC technique, as described in Section 3.3, in the following manner: The normalized values h_j were calculated using Equations (17) and (18); S_i and S_{ij}' were determined using Equations (19) and (20); the criterion removal effect was computed using Equation (21); and the weights of the criteria w_j were calculated using Equation (22). Section 3.2 offers instructions on how to effectively use the SAW approach to address the MCDM difficulty. Begin by calculating the decision-making matrices using Equation (13). The initial matrix should be normalized using formulas (14) and (15). Subsequently, the computation of V_i is performed utilizing formula (16). Ultimately, arrange the choices in a manner so the most favorable alternative possesses the highest V_i. The outcomes of the option ranking and parameter computation utilizing the SAW approach are presented in Table 4 (for the ultimate iteration of SAW analysis). According to the table, option 5 is the most optimal decision out of all the accessible possibilities. The optimal values for the fundamental design characteristics are u_1 = 3.62, X_{ba1} = 0.25, and X_{ba2} = 0.4, as specified in Table 5.

Figure 8 depicts the relationship between the ideal values of u_1 and u_{gb}. The following regression equation, which has an R^2 value of 0.9938, can be used to determine the optimal values of u_1:

$$u_1 = 0.1055 \cdot u_{gb} + 1.448 \qquad (23)$$

Table 3. Input design factors and output results for u_{gb} = 20

Trial.	u_1	X_{ba1}	X_{ba2}	L_{gb} (dm)	η_{gb} (%)
1	3.62	0.25	0.25	54.85	95.78
2	3.62	0.25	0.29	53.08	96.05
3	3.62	0.25	0.33	51.60	96.10
4	3.62	0.25	0.36	50.33	96.07
5	3.62	0.25	0.40	49.22	96.05
6	3.62	0.29	0.25	54.21	95.25
…					
35	3.63	0.29	0.40	48.57	95.53
36	3.63	0.33	0.25	53.66	94.64
37	3.63	0.33	0.29	51.89	94.68
…					
65	3.65	0.33	0.40	48.02	94.96
66	3.65	0.36	0.25	53.17	93.96
67	3.65	0.36	0.29	51.41	93.99
…					
98	3.66	0.40	0.33	49.51	93.41
99	3.66	0.40	0.36	48.24	93.45
100	3.66	0.40	0.40	47.14	93.49
…					
123	3.67	0.40	0.33	49.49	93.40
124	3.67	0.40	0.36	48.23	93.44
125	3.67	0.40	0.40	47.12	93.47

Table 4. Some calculated results and rankings of options by SAW for u_{gb} =20

Trial.	n_{ij}		V_i	Rank
	L_{gb}	η_{gb}		
1	0.8591	0.9967	0.9739	71
2	0.8877	0.9995	0.9810	46
3	0.9132	1.0000	0.9857	26
4	0.9362	0.9997	0.9892	11
5	0.9573	0.9995	0.9925	1
6	0.8692	0.9912	0.9710	91
…				
35	0.9701	0.9941	0.9901	7
36	0.8781	0.9848	0.9672	107
37	0.9081	0.9852	0.9725	87
…				
65	0.9813	0.9881	0.9870	19
66	0.8862	0.9777	0.9626	118
67	0.9166	0.9780	0.9679	103
…				
98	0.9517	0.9720	0.9687	99
99	0.9768	0.9724	0.9731	84
100	0.9996	0.9728	0.9773	63
…				
123	0.9521	0.9719	0.9686	100
124	0.9770	0.9723	0.9731	85
125	1	0.97263	0.9772	65

Table 5. Optimal main design factors

No.	u_{gb}					
	10	15	20	25	30	35
u_1	2.40	3.08	3.62	4.15	4.63	5.05
X_{ba1}	0.25	0.25	0.25	0.25	0.25	0.25
X_{ba2}	0.4	0.4	0.4	0.4	0.4	0.4

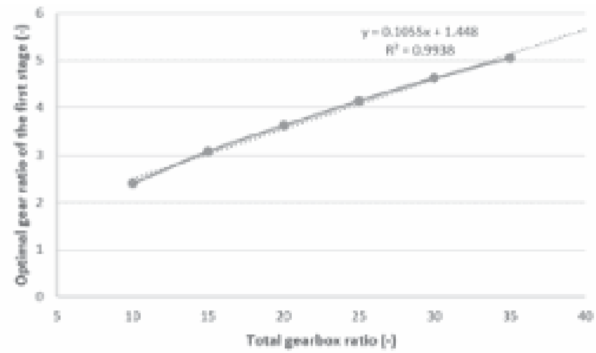


Figure 8. Relation between optimal values of u_1 and u_{gb}

6. CONCLUSIONS

The SAW technique was utilized in this study to deal with the MOOP of a two-stage helical gearbox. The objective of the study is to identify the optimal fundamental design factors that enhance gearbox efficiency while simultaneously reducing gearbox length. In order to accomplish this purpose, three crucial design characteristics, namely u_1, X_{ba1}, and X_{ba2}, were chosen. In addition, the SAW methodology was used to solve the MCDM problem, while the MEREC method was applied to calculate the weight of the criteria. The study's findings have helped determine the optimal primary design factors for a two-stage helical gearbox.

Acknowledgment:

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